Ratios, Proportions, and Percents

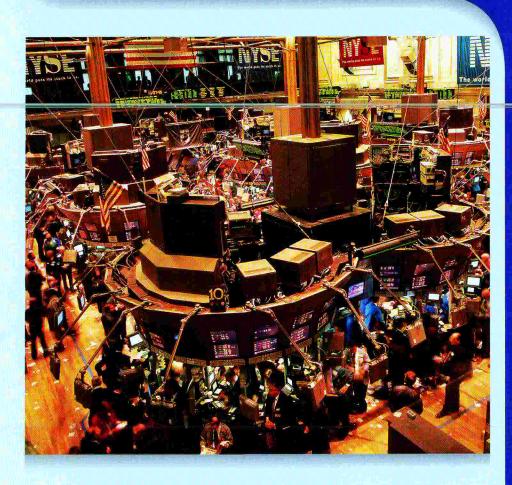




- 6-1 Ratios and Unit Rates
- 6-2 Proportions
- **6-3** Similar Figures and Scale Drawings
- 6-4 Probability
- **6-5** Fractions, Decimals, and Percents
- 6-6 Proportions and Percents
- 6-7 Percents and Equations
- 6-8 Percent of Change
- 6-9 Markup and Discount
- **6-10** Problem Solving: Make a Table

Key Vocabulary

- certain event (p. 306)
- commission (p. 321)
- complement (p. 306)
- cross products (p. 294)
- discount (p. 329)
- event (p. 305)
- impossible event (p. 306)
- indirect measurement (p. 300)
- markup (p. 329)
- odds (p. 307)
- outcomes (p. 305)
- percent (p. 310)
- percent of change (p. 325)
- probability (p. 305)
- proportion (p. 294)
- rate (p. 289)
- ratio (p. 288)
- scale drawing (p. 300)
- similar figures (p. 299)
- unit rate (p. 289)



Where You're Going

In this chapter, you will learn how to

- Find and use ratios and unit rates.
- Write and solve proportions.
- Find and use percents.
- Solve a problem by making a table.



Real-World Snapshots Applying what you learn, on pages 344–345 you will solve problems about stock trading.

Ratios and Unit Rates

What You'll Learn



To write and simplify ratios



To find rates and unit rates

... And Why

To solve real-world problems involving unit prices, gas mileage, and speed



- 1. $\frac{30}{35}$
- **2.** $\frac{24}{40}$
- **3.** $\frac{54}{60}$ **4.** $\frac{12}{15}$
- 5. $\frac{14}{42}$
- 6. $\frac{40}{24}$

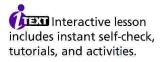


New Vocabulary

- ratio
- rate
- unit rate

Reading Math

Regardless of how you write the ratio, you read 10 to 15, 10 : 15 and $\frac{10}{15}$ as "ten to fifteen."



OBJECTIVE

Writing Ratios

Statistics In the United States, about 10 out of every 15 people eligible to vote are registered to vote. The numbers 10 and 15 form a ratio.

Key Concepts

Ratio

A ratio is a comparison of two quantities by division. You can write a ratio in different ways.

Arithmetic

10 to 15 $10:15 \quad \frac{10}{15}$

Algebra

a to b $a:b = \frac{a}{b}$, for $b \neq 0$

EXAMPLE

Real-World Problem Solving

Surveys A survey asked students whether they had after-school jobs. Write each ratio as a fraction in simplest form.

a. students with jobs to students without jobs

$$\frac{\text{students with jobs}}{\text{students without jobs}} = \frac{40}{60}$$

After-School Jobs

Response	Number
Have a Job	40
Don't Have a Job	60
Total	100

b. students without jobs to all students surveyed

$$\frac{\text{students without jobs}}{\text{all students surveyed}} = \frac{60}{100}$$

⋖ Check Understanding Example 1

- 1. Write each ratio as a fraction in simplest form.
 - a. students with jobs to all students surveyed
 - **b.** students without jobs to students with jobs

Finding Rates and Unit Rates

A rate is a ratio that compares quantities in different units. A unit rate is a rate that has a denominator of 1. Examples of unit rates include unit prices, gas mileage, and speed.

EXAMPLE

Real-World Problem Solving

Unit Prices The table shows prices for different sizes of the same dish detergent. Which size has the lowest unit price?

Regular:
$$\frac{\text{price}}{\text{volume}} \rightarrow \frac{\$1.20}{12 \text{ fl oz}} = \$.10/\text{fl oz}$$

Family:
$$\frac{\text{price}}{\text{volume}} \rightarrow \frac{\$2.24}{28 \text{ fl oz}} = \$.08/\text{fl oz}$$
 Find the unit prices

Economy:
$$\frac{\text{price}}{\text{volume}} \rightarrow \frac{\$3.60}{40 \text{ fl oz}} = \$.09/\text{fl oz}$$



Size	Volume (fl oz)	Price
Regular	12	\$1.20
Family	28	\$2.24
Economy	40	\$3.60

The family size has the lowest unit price.

Check Understanding Example 2

- 2. Find each unit rate.
 - a. Two liters of spring water cost \$1.98.
 - **b.** A car goes 425 mi on 12.5 gal of gas.

You can use dimensional analysis to choose conversion factors for converting rates.

EXAMPLE

Converting a Rate

Convert 10 mi/h to feet per minute.

$$10 \text{ mi/h} = \frac{10 \text{ mi}}{1 \text{ h}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{60 \text{ min}}$$
$$= \frac{110 \text{ mi}}{1 \text{ h}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{60 \text{ min}}$$
$$= \frac{880 \text{ ft}}{\text{min}}$$

Use conversion factors that convert miles to feet and hours to minutes.

Divide the common factors and units.

Simplify.

10 mi/h equals 880 ft/min.

Check Understanding Example 3

3. Complete each statement.

b.
$$12 \text{ cm/s} = \mathbf{m} \text{ m/h}$$

Practice and Problem Solving



Practice by Example

Write each ratio as a fraction in simplest form.

Example 1 (page 288)

- **1.** 9:27
- 2. 12 to 8
- **3.** 2 to 18
- 4.6:50

- 1,000 5. $\frac{1}{10,000}$
- 6.3:8
- **7.** 7 to 9
- 8. 8 out of 11
- **9.** 3 out of 12 people live in a rural area.
- **10.** 98 homes in 100 have a TV.
- **11.** 70 homes out of 125 have a personal computer.
- **12.** In one class, there are 6 girls for every 10 boys.

Example 2 (page 289)

Find each unit rate.

- **13.** A skydiver falls 144 ft in 3 s. **14.** A pump moves 42 gal in 7 min.
- **15.** A car travels 676 mi in 13 h. **16.** 20 c of water evaporate in 5 d.

Example 3 (page 289)

Complete each statement.

- 17. $720 \text{ m/day} = \mathbf{m} \text{ m/min}$
- **18.** 1.5 gal/min = \blacksquare qt/h
- **19.** 32 yd/min = 10 in./s
- **20.** 0.85 km/s = m/min
- **21.** 80 mi/h = \blacksquare ft/s
- **22.** 20 fl oz/min = \square qt/day

Apply Your Skills

Write each ratio as a fraction in simplest form.

- **23.** 36 to 48
- **24.** 60 to 24
- **25.** 16:12
- **26.** 15:27
- 🚱 27. Cycling Anna and Julia each take a bicycle trip. Anna rides 20 miles in $1\frac{1}{3}$ hours. Julia rides 246 miles in 16 hours. Which rider has the slower unit rate? By how much?
- 28. Transportation What is the rate in meters per second of a jetliner that is traveling at a rate of 846 km/h?
 - 29. Error Analysis A student converts 100 ft/min to 500 in./s. Use dimensional analysis to explain why the student's result is not reasonable.

Boys in Two Classes

Class	Number of Boys	Number of Students
Α	6	30
В	4	24

Use the table at the left for Exercises 30 and 31.

- 30. For each class, write the ratio of the number of boys to the total number of students.
- **31.** Which class has the greater ratio of boys to students?



32. Writing in Math A student claims that a ratio remains unchanged if 1 is added to both the numerator and the denominator of the fraction. Does $\frac{a}{b}$ equal $\frac{a+1}{b+1}$? Write an explanation, and give an example or a counterexample.

33. Science Density is the ratio of a substance's mass to its volume. A volume of 20 cubic centimeters of gold has a mass of 386 grams. Express the density of gold as a unit rate.



Test Prep

Multiple Choice

34. A 50-lb bag of Glossy Coat Horse Feed costs \$23.50. A 25-lb bag costs \$15.50. How much money per pound would you save by buying the bag with the lower unit price?

A. \$.15

B. \$.32

C. \$.47

D. \$.62

35. Karla and her dad were nailing up plywood. They started at 10:00. Karla drove 30 nails in 10 min, the time it took her dad to drive 50 nails. At that rate for each, when did they finish driving 392 nails in all?

F. 10:30

G. 10:39

H. 10:45

I. 10:49

Reading Comprehension

Read the passage below before doing Exercises 36–38.



A Sappy Story

Connecticut has more than 100 farms that produce maple syrup. Sugarers collect sap and boil it down to syrup. In a good year, one small sugarer in Connecticut

averages 301 gallons of sap weekly from 200 trees. The sap boils down to just seven gallons of syrup. The syrup sells for \$4.50 per half pint or \$44 per gallon.



- **36.** Write the ratio of sap to syrup in simplest form.
- 37. Find the unit prices for syrup sold by the half pint and syrup sold by the gallon. Which has the lower unit price?
- **38.** If the sugarer sells the syrup by the half pint, how much income will there be for 10 weeks of sugaring in a good year?

Mixed Review

Simplify each expression. Lesson 5-9

39.
$$(-3 \cdot 4)^3$$

40. $(2x^2y)^4$

41. $\left(-\frac{ab^3}{a^2b}\right)^3$

Compare. Use >, <, or = to complete each statement. Lesson 5-2

42.
$$\frac{7}{8} = \frac{14}{24}$$

42.
$$\frac{7}{8} = \frac{14}{24}$$
 43. $\frac{4}{12} = \frac{10}{30}$ **44.** $\frac{13}{20} = 0.6$

44.
$$\frac{13}{20} \blacksquare 0.6$$

Lesson 2-1 🚳 45. Vacation Three friends shared the driving on a long trip. Marla drove 7 mi more than Guido. Guido drove five times as far as Juanita did. Juanita drove 112 mi. How long was the trip?

Converting Between Measurement Systems

For Use With Lesson 6-1

You can use conversion factors (dimensional analysis) to convert a unit of measure from one system to another. For example, since 1 mi \approx 1.61 km, you can use $\frac{1 \text{ mi}}{1.61 \text{ km}}$ and $\frac{1.61 \text{ km}}{1 \text{ mi}}$ as conversion factors.

The table shows some useful conversion factors.

Customary Units and Metric Units	Conversion Factors
1 in. = 2.54 cm	$\frac{1 \text{ in.}}{2.54 \text{ cm}} \text{ or } \frac{2.54 \text{ cm}}{1 \text{ in.}}$
1 mi ≈ 1.61 km	$\frac{1 \text{ mi}}{1.61 \text{ km}} \text{ or } \frac{1.61 \text{ km}}{1 \text{ mi}}$
$1.06 \text{ qt} \approx 1 \text{ L}$	$\frac{1.06 \text{ qt}}{1 \text{ L}}$ or $\frac{1 \text{ L}}{1.06 \text{ qt}}$
1 oz ≈ 28.4 g	$\frac{1 \text{ oz}}{28.4 \text{ g}} \text{ or } \frac{28.4 \text{ g}}{1 \text{ oz}}$
$2.20 \text{ lb} \approx 1 \text{ kg}$	$\frac{2.20 \text{ lb}}{1 \text{ kg}}$ or $\frac{1 \text{ kg}}{2.20 \text{ lb}}$

In general, a conversion between systems results in an approximate measurement.

1 EXAMPLE

The longest track event at the Olympics is the 50-km walk. How long is the race in miles?

$$\begin{array}{ll} 50~km \approx 50~km \cdot \frac{1~mi}{1.61~km} & \text{Use a conversion factor that changes} \\ = 50~km \cdot \frac{1~mi}{1.61~km} & \text{Divide the common units.} \end{array}$$

$$= \frac{50 \text{ mi}}{1.61}$$
 Multiply.
$$\approx 31 \text{ mi}$$
 Divide.

The 50-km walk is about 31 mi long.

You can round within a conversion factor to get compatible numbers.

2 EXAMPLE

About how many ounces are in 60 grams?

$$60~g \approx 60~g \cdot \frac{1~oz}{28.4~g}$$
 Use the conversion factor that changes grams to ounces.

$$\approx 60~g \cdot \frac{1~oz}{30~g} \qquad \begin{array}{c} \text{Round within the conversion factor to} \\ \text{a number compatible with 60.} \end{array}$$

$$=60^{2}$$
g $\cdot \frac{1 \text{ oz}}{130 \text{ g}}$ Divide the common factors and units.

There are about 2 ounces in 60 grams.

Sometimes you may need to use two or more conversion factors.

3 EXAMPLE

A punch recipe calls for a gallon of sparkling water. How many 2-L bottles should you buy?

$$1 \text{ gal} \approx 1 \text{ gal} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} \cdot \frac{1 \text{ L}}{1.06 \text{ qt}}$$

Use conversion factors that change gallons to quarts and quarts to liters.

$$= 1 \text{ gal} \cdot \frac{4 \text{ qf}}{1 \text{ gal}} \cdot \frac{1 \text{ L}}{1.06 \text{ qf}}$$

Divide the common units.

$$= \frac{4 L}{1.06}$$

Multiply.

Divide.

Now find the number of bottles you need for 3.8 L.

$$\frac{3.8}{2} = 1.9$$

Divide by 2, since there are 2 L per bottle.

You need about 1.9 bottles. You should buy two bottles.

EXERCISES

Convert. Where necessary, round to the nearest tenth.

2.
$$16 \text{ cm} \approx \square \text{ in.}$$

3.
$$\blacksquare$$
 mi \approx 20 km

4.
$$\parallel$$
 km $\approx 100 \text{ mi}$

5.
$$\blacksquare$$
 L \approx 50 qt

6.
$$\blacksquare$$
 g \approx 15 oz

7.
$$15 L \approx \square qt$$

8.
$$\blacksquare$$
 lb \approx 14 kg

9.
$$44 \text{ lb} \approx 100 \text{ kg}$$

10.
$$100 \text{ oz} \approx 100 \text{ kg}$$

11.
$$\blacksquare$$
 L \approx 212 pt

12.
$$500 g \approx 110 lb$$

13.
$$1,000 \text{ mm} \approx 1000 \text{ in.}$$

14.
$$\blacksquare$$
 gal $\approx 20 \text{ L}$

- - 17. Writing in Math Explain how you would estimate the number of kilometers in 19 miles.
 - **18.** In Exercise 15, you may have found that $10 \text{ mi/h} \approx 16.1 \text{ km/h}$. Also, 10 mi/h = 880 ft/min (Example 3, p. 289). Convert both 16.1 km/h and 880 ft/min to meters per second and compare.
 - 19. Restate Exercise 27 on page 290 in equivalent metric units and solve.
 - **20.** In Exercise 28 on page 290, you convert 846 km/h to meters per second. Convert 846 km/h to miles per hour and then to feet per second.

Proportions

What You'll Learn



To solve proportions



To use proportions to solve problems

... And Why

To solve real-world problems involving science



Solve each equation.

1.
$$4x = 52$$
 2. $3y = 18$

2.
$$3y = 18$$

$$3.5b = 75$$

3.
$$5b = 75$$
 4. $7k = 21$

For help, go to Lesson 2-6.

New Vocabulary

- proportion
- cross products

OBJECTIVE

Solving Proportions

A **proportion** is an equality of two ratios—for example, $\frac{6}{9} = \frac{8}{12}$. You can use the Multiplication Property of Equality to show an important property of all proportions.

If
$$\frac{a}{b} = \frac{c}{d}$$

then $\frac{a}{b} \cdot bd = \frac{c}{d} \cdot bd$ Multiplication Property of Equality

$$\frac{ab^1d}{b} = \frac{cbd^1}{b}$$

$$\frac{ab^{1}d}{{}_{1}b} = \frac{cbd^{1}}{{}_{1}d} \qquad \frac{b}{b} = 1 \text{ and } \frac{d}{d} = 1$$

and ad = cb, or ad = bc.

The products ad and bc are called the cross products of the proportion $\frac{a}{b} = \frac{c}{d}$.

Key Concepts

Cross Products

In a proportion, the cross products are equal.

Arithmetic



$$6 \cdot 12 = 9 \cdot 8 = 72$$

Algebra



$$ad = bc$$

To solve a proportion that contains a variable, you find the value that makes the equation true.

Reading Math Read the proportion

 $\frac{6}{9} = \frac{8}{12}$ as "the ratio 6 to 9 equals the ratio 8 to 12," or as "6 is to 9 as 8 is to 12."

EXAMPLE

Multiplying to Solve a Proportion

Solve $\frac{x}{9} = \frac{4}{6}$.

Method 1 Multiplication Property of Equality

$$\frac{x}{9} = \frac{4}{6}$$

$$\frac{x}{9} \cdot 9 = \frac{4}{6} \cdot 9$$

$$x = \frac{36}{6}$$

$$r = 6$$

Method 2 Cross products

$$\frac{x}{9} = \frac{4}{6}$$

$$x \cdot 6 = 9 \cdot 4$$

$$6x = 36$$

$$\frac{6x}{6} = \frac{36}{6}$$

$$x = 6$$

Check Understanding Example 1

1. Solve each proportion.

a.
$$\frac{h}{9} = \frac{2}{3}$$

b.
$$\frac{4}{5} = \frac{t}{55}$$

b.
$$\frac{4}{5} = \frac{t}{55}$$
 c. $\frac{22}{d} = \frac{6}{21}$

Two ratios form a proportion if their cross products are equal.

EXAMPLE Testing for a Proportion

Do the ratios $\frac{4}{6}$ and $\frac{10}{14}$ form a proportion? Explain.

$$\frac{4}{6} \stackrel{?}{=} \frac{10}{14}$$
 Test by writing as a proportion.

$$4 \cdot 14 \stackrel{?}{=} 6 \cdot 10$$
 Write cross products.

$$56 \neq 60$$
 Simplify.

The ratios do not form a proportion. Cross products are not equal.

✓ Check Understanding Example 2

2. Tell whether the two ratios form a proportion. Explain.

a.
$$\frac{6}{9}, \frac{4}{6}$$

b.
$$\frac{15}{20}$$
, $\frac{5}{7}$

c.
$$\frac{7}{12}$$
, $\frac{17.5}{30}$

OBJECTIVE

Using Proportions to Solve Problems

You can write and solve proportions for many real-world problems.

Real-World Problem Solving EXAMPLE

Navigation One hundred nautical miles equals about 115 standard, or statute, miles. To the nearest mile, how far in statute miles is 156 nautical miles?

Let d = distance in statute miles.

$$\frac{\text{distance in nautical miles}}{\text{distance in statute miles}} \xrightarrow{} \frac{100}{115} = \frac{156}{d} \xleftarrow{} \frac{\text{distance in nautical miles}}{\text{distance in statute miles}}$$

$$100d = 115(156)$$
 Write cross products.

$$d = \frac{115(156)}{100}$$

Divide each side by 100.

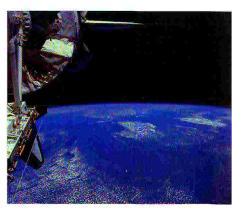
$$d \approx 179$$

A calculator may be useful.

156 nautical miles is about 179 statute miles.

Check Understanding Example 3

3. To the nearest mile, how far in nautical miles is 100 statute miles?



Real-World Connection

Sailors and astronauts measure distances in nautical miles. This photo of the Great Lakes was taken from the space shuttle at an altitude of 156 nautical miles.

Practice and Problem Solving

Practice by Example

Solve each proportion.

Example 1 (page 294)

$$\frac{2}{v} = \frac{1}{8}$$
 2. $\frac{z}{42} =$

$$\frac{z}{42} = \frac{25}{70}$$
 3. $\frac{4}{h}$

1.
$$\frac{2}{v} = \frac{1}{8}$$
 2. $\frac{z}{42} = \frac{25}{70}$ **3.** $\frac{4}{h} = \frac{8}{10}$ **4.** $\frac{4}{16} = \frac{s}{8}$

5.
$$\frac{4}{11} = \frac{x}{22}$$
 6. $\frac{2}{9} = \frac{r}{36}$ **7.** $\frac{12}{n} = \frac{2}{12}$ **8.** $\frac{1}{15} = \frac{3}{p}$

6.
$$\frac{2}{9} = \frac{r}{36}$$

7.
$$\frac{12}{n} = \frac{2}{12}$$

8.
$$\frac{1}{15} = \frac{3}{4}$$

9.
$$\frac{4}{15} = \frac{a}{75}$$

10.
$$\frac{3}{4} = \frac{21}{b}$$

9.
$$\frac{4}{15} = \frac{a}{75}$$
 10. $\frac{3}{4} = \frac{21}{b}$ **11.** $\frac{13}{6} = \frac{39}{60}$ **12.** $\frac{3}{6} = \frac{7}{d}$

12.
$$\frac{3}{4} = \frac{7}{4}$$

Example 2 (page 295)

Tell whether the two ratios form a proportion. Explain.

13.
$$\frac{2}{3}$$
 and $\frac{10}{20}$ **14.** $\frac{25}{80}$ and $\frac{5}{16}$ **15.** $\frac{4}{7}$ and $\frac{20}{25}$ **16.** $\frac{2}{3}$ and $\frac{10}{16}$

14.
$$\frac{25}{80}$$
 and $\frac{5}{16}$

15.
$$\frac{4}{7}$$
 and $\frac{20}{25}$

16.
$$\frac{2}{3}$$
 and $\frac{10}{16}$

17.
$$\frac{3}{4}$$
 and $\frac{12}{15}$

18.
$$\frac{3}{8}$$
 and $\frac{21}{56}$

17.
$$\frac{3}{4}$$
 and $\frac{12}{15}$ **18.** $\frac{3}{8}$ and $\frac{21}{56}$ **19.** $\frac{9}{24}$ and $\frac{15}{40}$ **20.** $\frac{20}{32}$ and $\frac{12}{20}$

20.
$$\frac{20}{32}$$
 and $\frac{12}{20}$

(page 295)

- Example 3 **21.** Photocopies At the Copy Shoppe, 18 copies cost \$1.08. At that rate, how much will 40 copies cost?
 - 22. Three tea bags are needed to make a gallon of iced tea. How many tea bags are needed to make four gallons?
 - 323. Purchasing Three posters cost \$9.60. At that rate, how many posters can you buy for \$48?

Apply Your Skills

Tell whether the two ratios form a proportion. Explain.

24.
$$\frac{3.9}{5.4}$$
 and $\frac{13}{18}$

25.
$$\frac{54}{60}$$
 and $\frac{118}{110}$

26.
$$\frac{27}{72}$$
 and $\frac{48}{128}$

24.
$$\frac{3.9}{5.4}$$
 and $\frac{13}{18}$ **25.** $\frac{54}{60}$ and $\frac{118}{110}$ **26.** $\frac{27}{72}$ and $\frac{48}{128}$ **27.** $\frac{144}{120}$ and $\frac{75}{145}$

Mental Math Solve by mental math.

28.
$$\frac{1}{6} = \frac{a}{72}$$

29.
$$\frac{120}{24} = \frac{3}{2}$$

28.
$$\frac{1}{6} = \frac{a}{72}$$
 29. $\frac{120}{24} = \frac{y}{2}$ **30.** $\frac{10}{v} = \frac{3}{1.5}$ **31.** $\frac{n}{12} = \frac{12}{2}$

31.
$$\frac{n}{12} = \frac{12}{2}$$

- 32. Exchange Rates On a recent day, the exchange rate for U.S. dollars to European euros was 0.89 dollar per euro. On that day, about how many euros would you get for 25 dollars?
 - 33. Error Analysis Fancy ribbon costs \$3 for 15 in. Your friend wants to find the cost of 3 ft of ribbon. He uses the proportion $\frac{3}{15} = \frac{x}{3}$ and gets an answer of \$.60. Explain your friend's error.

Solve each proportion. Where necessary, round to the nearest tenth.

34.
$$\frac{4}{3} = \frac{b}{21}$$

34.
$$\frac{4}{3} = \frac{b}{21}$$
 35. $\frac{6}{25} = \frac{e}{80}$ **36.** $\frac{4}{9} = \frac{f}{15}$ **37.** $\frac{3}{8} = \frac{50}{g}$

36.
$$\frac{4}{9} = \frac{f}{15}$$

37.
$$\frac{3}{8} = \frac{50}{g}$$

38.
$$\frac{24}{17} = \frac{109}{h}$$
 39. $\frac{7}{9} = \frac{j}{22.5}$ **40.** $\frac{6}{13} = \frac{7.8}{m}$ **41.** $\frac{20}{27} = \frac{1.1}{n}$

39.
$$\frac{7}{9} = \frac{j}{22.5}$$

40.
$$\frac{6}{13} = \frac{7.8}{m}$$

41.
$$\frac{20}{27} = \frac{1.1}{n}$$

Estimation Estimate the solution of each proportion.

42.
$$\frac{11}{a} = \frac{9}{17}$$

42.
$$\frac{11}{a} = \frac{9}{17}$$
 43. $\frac{w}{20} = \frac{6}{23}$ **44.** $\frac{3}{2} = \frac{29}{d}$ **45.** $\frac{20}{3.9} = \frac{s}{6}$

44.
$$\frac{3}{2} = \frac{29}{d}$$

45.
$$\frac{20}{3.9} = \frac{s}{6}$$

46.
$$\frac{1.5}{p} = \frac{2.1}{4.1}$$
 47. $\frac{f}{4} = \frac{12}{49}$ **48.** $\frac{60}{g} = \frac{24.1}{8.1}$ **49.** $\frac{9}{4.4} = \frac{x}{19}$

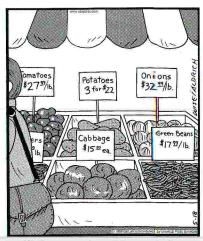
47.
$$\frac{f}{4} = \frac{12}{49}$$

48.
$$\frac{60}{g} = \frac{24.1}{8.1}$$

49.
$$\frac{9}{4.4} = \frac{x}{19}$$

- **50.** At the rate shown in the cartoon, how much would five potatoes cost?
- St. Quality Control A microchip inspector found three defective chips in a batch containing 750 chips. At that rate, how many defective chips would there be in 10,000 chips?
 - **52.** Reasoning If $\frac{a}{b} = \frac{c}{d}$, will $\frac{a}{c} = \frac{b}{d}$? Assume that $b \neq 0$, $c \neq 0$, and $d \neq 0$. Explain your reasoning.
 - **53. Geometry** A rectangle that is 20 cm long and 28 cm wide is the same shape as one that is 9 cm long and z cm wide. Find z.

REAL LIFE ADVENTURES by Gary Wise and Lance Aldrich



If the people who own the shops at the airport owned other things.

§ 54. Baseball Your team scores 4 runs in the first three innings of a 9-inning baseball game. If it continues at that rate, how many runs will it score in the game?

Write a proportion for each situation. Then solve.

- **55.** 3 oz for \$1.65; 5 oz for x dollars
- **56.** 20 lb for \$27.50; 12 lb for x dollars
- **57.** 25 yd in $2\frac{1}{2}$ s; 100 yd in x seconds
- **58.** 3 miles in 2.8 minutes; 33.3 miles in x minutes
- **59.** $3\frac{1}{2}$ pounds in 4 cubic inches; x pound in 1 cubic inch
- 60. Writing in Moth A truck driver estimates that it will take him 12 h to drive 1,160 km. After 5 h, he has driven 484 km. Is he on schedule? Explain.

Challenge

For Exercises 61-64, use the table.

- 61. How many times does an adult's heart beat in 270 s?
- 62. In how many seconds will a newborn's heart beat 35 times?
- 63. In how many seconds will a 12-year-old's heart beat 17 times?
- **64.** In 45 s, how many more times does a newborn's heart beat than a 6-year-old's heart?

Human Heart Rates

Age (years)	Beats per Minute
newborn	140
1	120
6	100
10	90
12	85
adult	80

Write a proportion for each situation. Then solve.

- **65.** 5 km in 18 min 36 s; 8 km in v minutes
- **66.** 96 oz for \$2; y pounds for \$10
- **67.** 4 oz for \$1.85; 1 lb for t dollars
- **68.** \$5.76 for 2 lb 4 oz; c dollars for 1 pound



Test Prep

Multiple Choice

- **69.** Four ounces of orange juice contain 50 calories. About how many calories are in 14 ounces of orange juice?
 - **A**. 1 cal
- **B.** 14 cal
- **C.** 175 cal
- **D.** 700 cal
- 70. A lion's heart beats 12 times in 16 s. How many times does a lion's heart beat in 60 s?
 - F. 24
- **G**. 32
- H. 45
- I. 192
- 71. An artist makes purple paint by mixing red and blue paint in the ratio of 2 parts red to 3 parts blue. What is the ratio of red paint to purple paint?
 - **A.** 3:2
- **B.** 3:5
- C. 2:3
- D. 2:5

Extended Response

- 72. On Monday, the ratio of Tara's pocket money to her brother Seth's pocket money was $\frac{3}{1}$. On Tuesday, Tara gave \$5 to Seth. Then Tara had twice as much money as Seth. Let 3x equal the amount Tara had on Monday and x equal the amount Seth had on Monday.
 - a. Write two ratios that each compare the amount of money Tara had on Tuesday to the amount Seth had on Tuesday. Use the ratios to write a proportion.
 - **b.** Solve for x.
 - c. Find the amount of money each person had on Monday.



Mixed Review

Lesson 6-1 Write each ratio as a fraction in simplest form.

- **73.** ten per thousand **74.** 30 to 55
- **75.** 125:70
- Lesson 5-6 **76.** Personal Finance On Saturday afternoon, a student bought two music tapes for \$8.95 each and a sweater for \$24.95. She received \$20 for moving a lawn. On Saturday night, she had \$45.12. How much money did the student have on Saturday morning?

Lessons 1-3 and 5-3 Tell whether each equation is true or false.

77.
$$\left| -2\frac{1}{4} \right| - \left| 2\frac{1}{4} \right| = 0$$
 78. $\left| -2\frac{1}{4} \right| + \left| 2\frac{1}{4} \right| = 0$ 79. $-\left| -\frac{9}{4} \right| + \left| 2\frac{1}{4} \right| = 0$ 80. $\left| -\frac{9}{4} \right| - \left| 2\frac{1}{4} \right| = 0$

78.
$$\left| -2\frac{1}{4} \right| + \left| 2\frac{1}{4} \right| = 0$$

79.
$$-\left| -\frac{9}{4} \right| + \left| 2\frac{1}{4} \right| = 0$$

80.
$$\left| -\frac{9}{4} \right| - \left| 2\frac{1}{4} \right| = 0$$

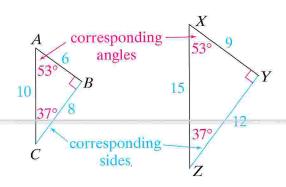
Similar Figures and **Scale Drawings**

OBJECTIVE

Using Similar Figures

Similar figures have the same shape, but not necessarily the same size. Similar figures have corresponding angles and corresponding sides.

The symbol \sim means is similar to. At the right, $\triangle ABC \sim \triangle XYZ$.



Key Concepts

Similar Figures

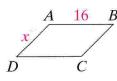
Similar figures have two properties.

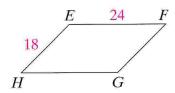
- The corresponding angles have equal measures.
 - The lengths of corresponding sides are in proportion.

EXAMPLE

Using Similar Figures

Parallelogram $ABCD \sim$ parallelogram EFGH. Find the value of x.





Write a proportion for corresponding sides.

Side DA corresponds to side HE.

$$\frac{x}{18} = \frac{16}{24}$$

Side AB corresponds to side EF.

$$x \cdot 24 = 18 \cdot 16$$

Write cross products.

$$\frac{24x}{24} = \frac{18 \cdot 16}{24}$$

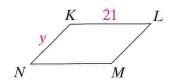
Divide each side by 24.

x = 12

Simplify.

Check Understanding Example 1

1. Parallelogram *KLMN* is similar to parallelogram ABCD in Example 1. Find the value of y. Round to the nearest tenth.



What You'll Learn



To solve problems that involve similar figures



To solve problems that involve scale drawings

... And Why

To solve real-world problems involving maps

Check Skills You'll Need

Solve each proportion. Round to the nearest tenth where necessary.

1.
$$\frac{2}{3} = \frac{f}{21}$$
 2. $\frac{3}{8} = \frac{50}{p}$

3.
$$\frac{9}{4} = \frac{15}{p}$$
 4. $\frac{16}{3} = \frac{19}{g}$

4.
$$\frac{16}{3} = \frac{19}{g}$$

For help, go to Lesson 6-2.

New Vocabulary

- similar figures
- indirect measurement
- scale drawing

Interactive lesson includes instant self-check, tutorials, and activities.

You can use similar figures to compute distances that are difficult to measure directly. Such a process is called **indirect measurement**.

2 EXAMPLE

Real-World Problem Solving

Indirect Measurement A tree casts a shadow 10 ft long. A 5-ft woman casts a shadow 4 ft long. The triangle shown for the woman and her shadow is similar to the triangle shown for the tree and its shadow. How tall is the tree?

10 ft

$$\frac{4}{10} = \frac{5}{x}$$
 Corresponding sides of similar triangles are in proportion.

$$4x = 10 \cdot 5$$
 Write cross products.

$$\frac{4x}{4} = \frac{10 \cdot 5}{4}$$
 Divide each side by 4.

$$x = 12.5$$
 Simplify.

The tree is 12.5 ft tall.

Check Understanding Example 2

2. Indirect Measurement A building 70 ft high casts a 150-ft shadow. A nearby flagpole casts a 60-ft shadow. Draw a diagram. Use similar triangles to find the height of the flagpole.

OBJECTIVE

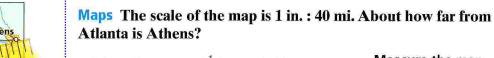
2/1

Using Scale Drawings

A **scale drawing** is an enlarged or reduced drawing that is similar to an actual object or place. The ratio of a distance in the drawing to the corresponding actual distance is the *scale* of the drawing.



Real-World Problem Solving



Map distance =
$$1\frac{1}{2}$$
 in., or 1.5 in.

$$\frac{\text{map (in.)}}{\text{actual (mi)}} \xrightarrow{\longrightarrow} \frac{1}{40} = \frac{1.5}{d} \xrightarrow{\longleftarrow} \frac{\text{map (in.)}}{\text{actual (mi)}}$$

$$1 \cdot d = 40 \cdot 1.5$$

$$1 \cdot d = 40 \cdot 1.5$$
 Write cross products. $d = 60$ Simplify.

Athens is about 60 mi from Atlanta.



3. Maps The distance from Atlanta to Macon is about 75 mi. What is the approximate map distance between these two cities?

Macon

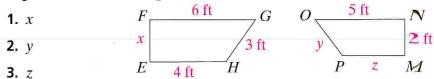
Practice and Problem Solving



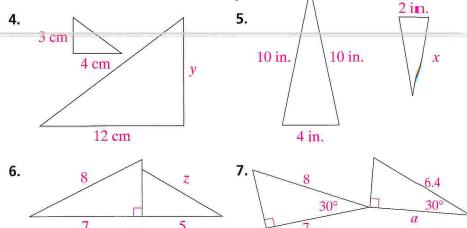
Practice by Example

Trapezoid $EFGH \sim \text{trapezoid } MNOP$. Find the indicated value.

Example 1 (page 299)



The triangles in each pair are similar. Find the missing length. Round to the nearest tenth where necessary. $\quad \ \ \, \bigwedge$



Example 2 (page 300)

- **8.** Indirect Measurement A tree casts a shadow 8 ft long. A 6-ft man casts a shadow 4 ft long. The triangle formed by the tree and its shadow is similar to the triangle formed by the man and his shadow. How tall is the tree?
- 9. Projection An image on a slide is similar to its projected image. A slide is 35 mm wide and 21 mm high. Its projected image is 85 cm wide. To the nearest centimeter, how high is the image?

Example 3 (page 300)

The scale of a map is 1 cm: 12 km. Find the actual distance for each map distance.

- **10.** 1.5 cm
- **11.** 12 cm
- **12.** 4.25 cm
- **13.** 8.3 cm
- 14. Maps Duane is drawing a map with a scale of 1 in.: 3 mi. He knows that the distance from Center Point to Comfort is 9 miles. How far apart should Duane locate the two towns on his map?

The scale of a drawing is 1 in.: 25 yd. Find the length on the drawing for each actual length.

- **15.** 100 yd
- **16.** 375 yd
- **17.** 512.5 yd
- **18.** 20 yd
- **19. Indirect Measurement** Jacques has a scale drawing of his bedroom with a scale of 1 cm: 0.4 m. On the drawing, the front window is 3 cm from the door. What is the actual distance in the room?

20. A scale drawing has a scale of 1 in.: 10 ft. What is the distance on the drawing for an actual distance of 20 ft? Of 45 ft?



The scale of a map is 2 cm: 15 km. Find the actual distance for each map distance.

21. 6 cm

22. 2.1 cm

23. 10 mm

24. 17.4 cm



The length of each piece in a model railroad built on the HO scale is $\frac{1}{87}$ of the actual length. Another popular model is the N scale, for which the scale is $\frac{1}{160}$.

- 25. The student in the photograph is holding HO and N models of the same locomotive. Which type of model is labeled A? Which type of model is labeled B?
- **26.** Each car on a full-size passenger train is 80 ft long. What is the length in inches of a model passenger car in the HO scale? In the N scale?
- **27.** A diesel locomotive is 60 ft long. What is the length in inches of a model of the locomotive in the N scale?
- **28.** In the O scale, a length is $\frac{1}{48}$ the actual length. An O-scale locomotive is 1.05 ft long. How long is the actual locomotive?

A scale drawing has a scale of $\frac{1}{2}$ in.: 10 ft. Find the length on the drawing for each actual length.

29. 40 ft.

30. 5 ft

31. 35 ft

32. $3\frac{1}{2}$ ft

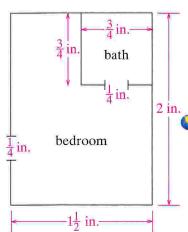
33. Open-Ended Give some examples of similar figures you find in everyday life.

The cities of Jackson, Mississippi, and Carson City, Nevada, are 1,750 mi apart.

- **34. Geography** A map of the United States has a scale of 1 in.: 250 mi. How far apart are the cities on the map?
 - **35.** On another map, the cities are 5 in. apart. What is the scale?
 - **36.** Reasoning A note at the bottom of a map says "not to scale." Explain why that is important information.

A 2-in. length in the scale drawing at the left represents an actual length of 20 ft.

- **37.** Architecture What is the scale of the drawing?
 - **38.** What are the actual dimensions of the bath?
 - **39.** Find the actual width of the doorways that lead into the bedroom and the bathroom.
 - **40.** Find the actual area of the bedroom.



- **41.** Can a bed 6 ft long and 3 ft wide fit into the narrow section of the bedroom? Justify your answer.
- Challenge 42. Architecture The length of a room is 16 ft. The scale of a blueprint is $\frac{1}{2}$ in. : 1 ft. Find the room's length in the blueprint.
 - 43. Writing in Math Explain why all squares are similar. For what other shape can you say that all figures are similar? Explain.
 - **44.** A boxcar on a freight train is 40 ft long. A model boxcar is 3 in. long. In which scale, HO, N, or O was the model built? (*Hint:* See Exercises 25–28.)
 - **45.** You are building a display shelf for your model train. You have 12 cars. Each car is 1.2 ft long. You want 1.2 in. of space between cars. How long must the shelf be?



Test Prep

Multiple Choice

- 46. The scale for a drawing of a garage is 1.5 in.: 12 ft. If one side of the garage is 40 ft, how long would that side be in the drawing? A. 5 in. **B.** 0.45 ft C. 5 ft D. 320 in.
- 47. The scale of a dollhouse is 1 in. : 2 ft. Which is most likely to be the measurement of the height of the dollhouse's front door?

F. $3\frac{1}{2}$ in.

G. $3\frac{1}{2}$ ft

H. 14 in.

Short Response

48. To plan a rectangular mural 90 ft long and 75 ft wide, you want to make a drawing with a scale of 1 in. : 9 ft. Can you fit the drawing on a piece of paper that is $8\frac{1}{2}$ in. by 11 in.? Explain.



49. Cheryl's goal is to ride 20 miles on a bike. The distance on a map from her house to the park is 4 in. The map scale is 1 in. : 2 mi. (a) Will Cheryl meet her goal if she rides from her house to the park and back? (b) Explain in words how you found your answer.

Mixed Review

Solve each proportion. Lesson 6-2

50.
$$\frac{x}{5} = \frac{32}{80}$$

51.
$$\frac{3}{8} = \frac{r}{14}$$

52.
$$\frac{40}{w} = \frac{50}{3}$$

50.
$$\frac{x}{5} = \frac{32}{80}$$
 51. $\frac{3}{8} = \frac{r}{15}$ **52.** $\frac{40}{w} = \frac{50}{3}$ **53.** $\frac{24}{16} = \frac{204}{c}$

- Lesson 6-1 6 54. Gas Mileage A car travels 264 mi on 12 gal of gas. Find the unit rate in miles per gallon.
- Write each fraction as a decimal. Lesson 5-2

55. $\frac{3}{8}$

56. $\frac{4}{9}$

57. $\frac{7}{16}$ **58.** $\frac{5}{12}$

Lesson 3-3 Find the mean (to nearest tenth), median, and mode.

59. 12, 10, 11, 7, 9, 8, 10, 5

60. 4.5, 3.2, 6.3, 5.2, 5, 4.8, 6, 3.9



Dilations

For Use With Lesson 6-3

You can use geometry software to make a scale drawing, or *dilation*, of a figure. First choose the Dilate command. Then choose a center of dilation and a scale, which is also known as a scale factor.

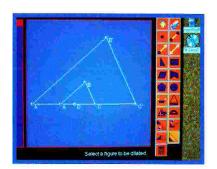
EXAMPLE

Draw a triangle. Then draw a dilation with scale factor 3.

Use geometry software. Draw $\triangle ABC$. Draw point D on one side of the triangle. Choose D as the center of a dilation with scale factor 3.

The result is an image like the one at the right. Each side of the dilation is 3 times as long as the corresponding side of $\triangle ABC$.

If you move point D, the dilation also will move. If instead you move A, B, or C, the dilation will change as $\triangle ABC$ changes.



EXERCISES

Use geometry software to draw $\triangle POR$.

- **1. a.** Draw a point S outside $\triangle PQR$. Draw a dilation of $\triangle PQR$ with center S and scale factor 2.5. Label the dilation $\triangle XYZ$. $\triangle XYZ$ is similar to $\triangle PQR$. Angle X corresponds to angle P, angle Y corresponds to angle Q, and angle Z corresponds to angle R.
 - **b.** Compare the location of $\triangle XYZ$ to the location of $\triangle PQR$. Does the dilation lie inside the original triangle? Outside the triangle? Do the triangles overlap?
 - **c.** Now move S to be *inside* $\triangle PQR$. Once again, compare the locations of the two triangles. How did moving the center of dilation change the relative locations of the triangles?
- **2.** Change the location of point S so that $\triangle PQR$ and $\triangle XYZ$ have the given number of points in common. Print an example of each case.
 - **a.** 0
- **b**. 1
- **c.** 2
- **d.** more than 2
- **3.** With S inside $\triangle PQR$, change the scale factor to 0.5. Describe the relative locations of the two triangles.
- **4. a.** Keep the scale factor of the dilation at 0.5. Use the Area tool to find the area of $\triangle PQR$. Use the Area tool again to find the area of $\triangle XYZ$. Write a ratio to compare the areas.
 - **b.** Move P, Q, or R to see how the area of $\triangle XYZ$ changes as the area of $\triangle PQR$ changes. Does the ratio of the areas change?
 - c. Reasoning What do your results suggest about the areas of similar triangles that have a scale factor of 0.5?



Probability

Sums of 2 Number Cubes

4

欄

7

8

9

m

6

7

8

9

10

6

m

8

9

10

11

7

8

9

10

12

OBJECTIVE

Finding Probability



Exploring Probability

Many board games involve rolling two number cubes and then adding the numbers on the cubés. Are certain sums more likely than others? The table shows the possible rolls and their sums.

- 1. Copy and complete the table.
- 2. What is the number of times each sum appears in the table?

1

2

3

4

5

6

2

3

4

5

6

3

4

5

7

8

- 3. Which sum appears most frequently?
- **4.** There is a total of 36 sums in the table. Use your answer to Question 3 to write the ratio number of times the most frequent sum appears total number of sums

Outcomes are the possible results of an action. There are six outcomes for rolling a single number cube: 1, 2, 3, 4, 5, and 6.

An event is any outcome or group of outcomes. The outcomes are called *favorable outcomes*. In rolling two number cubes, for example, rolling a sum of 4 is an event corresponding to the three favorable outcomes shown here.



Three outcomes result in the event a sum of 4.

The outcomes for rolling two number cubes are random and therefore equally likely to occur. When outcomes are equally likely, you can use a ratio to find the probability of an event.

probability of an event = $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$ number of possible outcomes

What You'll Learn



To find probability



To find odds

... And Why

To solve real-world problems involving the likelihood of events

Check Skills You'll Need Simplify.

1.
$$1-\frac{3}{8}$$

1.
$$1 - \frac{3}{8}$$
 2. $1 - \frac{17}{20}$

3.
$$1 - \frac{6}{11}$$
 4. $1 - \frac{1}{12}$

4.
$$1 - \frac{1}{12}$$



New Vocabulary

- outcomes
- event
- probability
- impossible event
- certain event
- complement
- odds

Interactive lesson includes instant self-check, tutorials, and activities.



Reading Math

You can read the probability 3 as "three in six" or "three out of six."



On a multiple-choice test item, you can eliminate any answer choice showing a probability greater than 1 because a probability cannot be greater than 1.

EXAMPLE

Finding Probability

Find P(rolling an even number) with one number cube.

$$\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{3}{6}$$

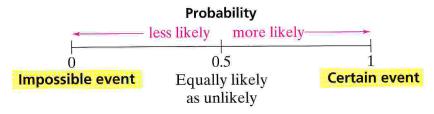
3 even-number outcomes

 $P(\text{rolling an even number}) = \frac{3}{6}, \text{ or } \frac{1}{2}.$

Check Understanding Example 1

- 1. Find each probability for one roll of a number cube.
 - **a.** *P*(odd number)
- **b.** P(2)
- **c.** P(5 or 6)

All probabilities range from 0 to 1.



The **complement** of an event is the opposite of that event. The events *no rain* and *rain* are complements of each other. The probability of an event plus the probability of its complement always equals 1.

EXAMPLE

Real-World Problem Solving

Vital Statistics In the United States, the probability that a child is a twin is 2 in 90, or $\frac{2}{90}$. Find P(not a twin).

$$P(twin) + P(not a twin) = 1$$

Write an equation.

$$\frac{2}{90} + P(\text{not a twin}) = 1$$

Substitute.

$$\frac{2}{90} - \frac{2}{90} + P(\text{not a twin}) = 1 - \frac{2}{90}$$
 Subtract $\frac{2}{90}$ from each side.

$$P(\text{not a twin}) = \frac{88}{90} = \frac{44}{45}$$

Simplify.

The probability that a child is not a twin is $\frac{44}{45}$.

✓ Check Understanding Example 2

- **2. a.** When you roll a number cube, what is P(not 2)?
 - b. Reasoning What is the complement of an impossible event?

OBJECTIVE

Finding Odds

You can think of probability as a ratio of $\frac{part}{whole}$. You can also use a part ratio, called odds, to describe the likelihood of an event.

odds in favor of an event = $\frac{\text{number of } favorable \text{ outcomes}}{\text{number of } unfavorable \text{ outcomes}}$

odds against an event = $\frac{\text{number of } unfavorable \text{ outcomes}}{\text{number of } favorable \text{ outcomes}}$

EXAMPLE

Real-World Problem Solving

Coins The reverse sides of five quarters are shown below. If you select one of these quarters at random, what are the odds in favor of it showing at least one human figure on its reverse side?



odds in favor = $\frac{3}{2}$ $\stackrel{\textstyle \longleftarrow}{\longleftarrow}$ 3 have a human figure. 2 do not.

The odds are $\frac{3}{2}$, or 3 to 2, in favor.



Reading Math

Read odds of $\frac{3}{2}$ as "three to two.

Check Understanding Example 3

- 3. You choose a quarter at random from the five above.
 - a. What are the odds in favor of it showing a horse?
 - b. What are the odds against it showing a horse?
 - c. Consider the event that the quarter shows the outline of a state.
 - i. What are the odds in favor of the event?
 - ii. What are the odds against the event?

Practice and Problem Solving



Practice by Example

Find each probability for one roll of a number cube.

Examples 1 and 2 (page 306) **1.** P(3)

2. P(3 or 4)

3. P(1, 2, or 3)

4. *P*(not 2, 3, or 6)

5. *P*(not 1, 3, 4, or 5)

6. P(less than 4)

Find each probability for selecting a letter at random from the word ARKANSAS.

7. P(A)

8. *P*(R)

9. P(S)

10. *P*(K or N)

11. *P*(vowel)

12. P(consonant)

- **13.** A box of crayons contains one crayon of each of the following colors: red, orange, yellow, green, blue, purple, black, white, pink.
 - **a.** What is the probability of NOT choosing a 6-letter color?
 - **b.** What is the complement of choosing green?

Example 3 (page 307) **14.** Suppose you choose a letter at random from the word ARITHMETIC. What are the odds in favor of selecting a vowel? What are the odds against selecting a vowel?

A teacher chooses a student at random from a class of 10 boys and 15 girls. Find the odds in favor of, and the odds against, each event.

15. choosing a girl

16. choosing a boy



Find each probability for one roll of a number cube.

17. *P*(7)

18. *P*(less than 3)

19. P(greater than 2)

Find each probability for choosing a letter at random from the word MATHEMATICS.

Lola's Socks

Color	Number of Socks
Pink	6
White	4
Green	3
Purple	2

20. *P*(K)

21. *P*(M, A, or T) **22.** *P*(vowel)

23. *P*(consonant)

24. Reasoning The table at the left describes the loose socks in Lola's drawer. One morning Lola pulls a sock from the drawer without looking. It is white. She pulls out another sock without looking. Find the probability that it also is white.

You have a set of 36 flash cards numbered from 1 to 36. A card is chosen at random. Find the odds in favor of, and the odds against, each selection.

25. even number

26. greater than 20

27. multiple of 3

28. prime number

29. multiple of 2 or 3 **30.** multiple of 2 and 3

31. It has only one digit.

32. It has more than one digit.

Challenge

33. Open-Ended Give an example of an event for which the probability equals 1. Justify your answer.

- **34. Error Analysis** Your friend is tossing a coin. He says that heads and tails are equally likely outcomes, so the probability of getting heads is $\frac{50}{50}$. Explain your friend's error.
- 35. Writing in Math Explain how you can use odds to find probability. Include an example.



Test Prep

Multiple Choice

- **36.** You draw a name at random from a hat holding the names of 6 girls and 8 boys. What are the odds in favor of choosing a boy?
 - A. 3 to 4
- **B.** 4 to 3
- C. 3 to 7
- D. 4 to 7



- 37. Refer to the spinner. What is the probability of the complement of stopping on either red or yellow?
 - F. 1
- G. ⅓
- H. 글
- 38. Suppose you roll a number cube. Which event has the same probability as P(not 1, 2, or 3)?
 - A. 3 or 4
- B. less than 5 C. not odd
- D. more than 4

Short Response

- 39. a. Can a probability be greater than 1?
 - **b.** Explain your answer.

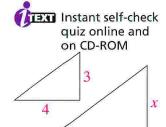
Mixed Review

- The scale of a map is 3 in.: 20 mi. Find the actual distance for each Lesson 6-3 map distance.
 - **40**. 6 in.
- **41.** 1 in.
- **42.** 4.2 in.
- **43.** $10\frac{1}{2}$ in.
- Write each decimal as a fraction or mixed number in simplest form. Lesson 5-2
 - **44.** 0.25
- **45.** 0.6
- **46.** 0.8125
- **47.** 5.15
- Lesson 2-6 🚇 48. Ticket Sales Students paid \$855 for tickets to a dance. Each ticket cost \$5. Write and solve an equation to find the number of tickets the students purchased.



Checkpoint Quiz 1

Lessons 6-1 through 6-4



6

- Write each phrase as a unit rate.
 - **1.** 20 mi in 5 h
- **2.** 42 gal in 7 min
- **3.** a fall of 144 ft in 3 s
- **4.** Geometry The figures (left) are similar. Find the missing length.
- 5. A person blinks 112 times in 4 min. At that rate, how many times does the person blink in 1.5 min?
- **6.** Suppose you roll a number cube. What is P(2 or 3)?



Fractions, Decimals, and Percents

What You'll Learn



To write percents as fractions and decimals



To write decimals and fractions as percents

... And Why

To solve real-world problems involving statistics



Check Skills You'll Need

Write each fraction as a decimal.

- 1. $\frac{5}{8}$
- **2.** $\frac{9}{20}$
- 3. $\frac{3}{4}$
- **4.** $\frac{5}{6}$
- **5.** $\frac{2}{3}$
- **6**. $\frac{8}{11}$



New Vocabulary

percent

OBJECTIVE



Writing Percents as Fractions and Decimals

A **percent** is a ratio that compares a number to 100. Therefore, you can write a percent as a fraction with a denominator of 100.

1 EXAMPLE

Writing a Percent as a Fraction

Write each percent as a fraction or a mixed number.

Write as a fraction with a denominator of 100.

$$\longrightarrow \frac{125}{100}$$

b. 125%

$$\frac{1}{20}$$
 \leftarrow

Simplify.
$$\longrightarrow \frac{5}{4}$$

Write as a mixed number. \longrightarrow $1\frac{1}{4}$

Check Understanding Example 1

- **1.** Write each percent as a fraction or mixed number in simplest form.
 - **a.** 58%
- **b.** 72%
- **c.** 144%

To write a percent as a decimal, write the percent as a fraction with a denominator of 100. Then divide to convert the fraction to a decimal.

Reading Math

Percent means "per hundred." The root cent shows up in many other words, such as centimeter, century, and centipede. In money, a cent is $\frac{1}{100}$ of a dollar, or \$.01.

2 EXAMPLE

Writing a Percent as a Decimal

Write 9.7% as a decimal.

$$9.7\% = \frac{9.7}{100}$$
$$= 009.7$$

Write as a fraction with a denominator of 100.

Divide by moving the decimal point left two places. You may need to write one or more zeros.

= 0.097

Check Understanding Example 2

- 2. Write each percent as a decimal.
 - **a.** 16%
- **b.** 62.5%
- **c.** 120%
- **d.** Biology About 45% of the people in the United States have type O blood. Write this percent as a decimal and as a fraction in simplest form.



Writing Decimals and Fractions as Percents

To write a decimal as a percent, rewrite the decimal as a fraction with a denominator of 100. Then write the fraction as a percent.

Another way to change a decimal to a percent is to move the decimal point two places to the right and add a percent sign.



Need Help?

For help on writing decimals as fractions, see Lesson 5-2.

EXAMPLE

Writing a Decimal as a Percent

Write 0.333 as a percent.

Method 1

Method 2

Rewrite as a fraction.

Move the decimal point.

$$0.333 = 33.3\%$$

$$0.333 = \frac{333}{1,000}$$

$$= \frac{333 \div 10}{1,000 \div 10}$$

$$= \frac{33.3}{100}$$

$$= 33.3\%$$

✓ Check Understanding Example 3

- 3. Write each decimal as a percent.
 - **a.** 0.4
- **b.** 0.023
- **c.** 1.75

To write a fraction as a percent, divide the numerator by the denominator. Then convert the decimal quotient to a percent.

EXAMPLE

Real-World Problem Solving

Pets Five out of sixteen families in the United States own dogs. What percent of families own dogs?

Write a fraction.

0.3125 Divide the numerator by the denominator.

31.25% Write as a percent.

About 31% of families own dogs.

Check Understanding Example 4

4. Three out of eleven families in the United States own cats. To the nearest percent, what percent of families own cats?



Real-World Connection There are about 55 million dogs and 61 million cats in the United States.

Practice and Problem Solving

Practice by Example

Write each percent as a fraction or mixed number in simplest form.

Example 1 (page 310)

6.6%

Example 2 (page 310) Write each percent as a decimal.

21. Education In 2000, women made up 40% of freshmen studying computer science at a certain university. Write this percent as a decimal and as a fraction in simplest form.

Example 3 (page 311) Write each decimal as a percent.

Example 4 (page 311)

Write each fraction as a percent. Round to the nearest tenth of a percent where necessary.

32.
$$\frac{23}{100}$$
 33. $\frac{1}{4}$ **34.** $\frac{11}{20}$ **35.** $\frac{3}{5}$ **36.** $\frac{5}{8}$

33.
$$\frac{1}{4}$$

34.
$$\frac{11}{20}$$

35.
$$\frac{3}{5}$$

36.
$$\frac{5}{8}$$

37.
$$\frac{4}{19}$$
 38. $\frac{1}{6}$ **39.** $\frac{7}{20}$ **40.** $\frac{2}{9}$ **41.** $\frac{7}{18}$

38.
$$\frac{1}{6}$$

39.
$$\frac{7}{20}$$

40.
$$\frac{2}{9}$$

41.
$$\frac{7}{18}$$

- **42. Populations** In the United States, about one person in eight lives in California. To the nearest percent, what percent of people in the United States live in California?
- 43. Homework Ron has read 14 pages of his 22-page reading assignment. To the nearest percent, what percent of the assignment has Ron read?

Apply Your Skills

Write each fraction as a percent. Round to the nearest tenth of a percent where necessary.

44.
$$\frac{8}{13}$$

45.
$$\frac{5}{2}$$

46.
$$\frac{11}{10}$$

45.
$$\frac{5}{6}$$
 46. $\frac{111}{100}$ **47.** $\frac{9}{2}$ **48.** $\frac{12}{5}$

Estimation About what percent of each flag is red?

49.





Probability Find each probability for one roll of a number cube. Write the probability as a percent. Round to the nearest tenth of a percent where necessary.

51. *P*(6)

52. *P*(even)

53. *P*(1 or 2)

54. *P*(not 1)

١t

m

Copy and complete the table.

	Fraction	Decimal	Percent
55.	<u>4</u> 5		
56.		0.10	
57.		0.5	
58.	3/4		
59.			67%
60.			25%



Real-World Connection Each year, about 45 million

Americans go camping.

Reasoning For Exercises 61–64, does each sentence make sense? Explain.

- **61.** About 17% of Americans go camping. That means about 83% do not go camping.
- **62.** A student correctly answered 200% of the items on a test.
- **63.** Today a runner ran 150% of the distance she ran yesterday.
- **64.** On a test, a student missed 12 items and correctly answered 96% of all items.

Compare. Use >, <, or = to complete each statement.

65.
$$0.05\% \equiv 50\%$$
 66. $\frac{7}{12} \equiv 60\%$

66.
$$\frac{7}{12} \blacksquare 60\%$$

68.
$$\frac{7}{8}$$
 \blacksquare 68%

69. 0.1756 ■ 176% **70.**
$$\frac{140}{130}$$
 ■ 104%

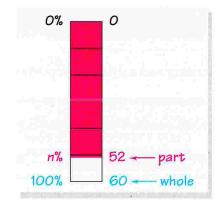
- **71.** Maps A map has a scale of 0.01%. Write the scale as a fraction.
 - **72.** Jeanette answered 32 questions correctly on a 45-question test. The passing grade was 70%. Did Jeanette pass? Justify your answer.
- **③ 73. Scale Drawings** A scale drawing has a scale of 1: 12. Write the scale as a percent.
 - 74. Writing in Math Explain how to write a decimal as a percent. Give examples.
- 🚱 **75. a. Test Grades** On his last math assignment, Kyle answered 5% of the questions incorrectly, or 1 question. How many questions did Kyle answer correctly?
 - **b.** On the same test, Diana answered 16 questions correctly. What percent of the questions did she not answer correctly?

31

2 EXAMPLE

Finding a Percent

What percent of 60 is 52? Round to the nearest tenth of a percent.



$$\frac{n}{100} = \frac{52}{60}$$

Write a proportion.

$$60n = 100(52)$$

Write cross products.

$$\frac{60n}{60} = \frac{100(52)}{60}$$

Divide each side by 60.

$$n = 86.\overline{6}$$

Simplify.

Round.

52 is approximately 86.7% of 60.

✓ Check Understanding Example 2

- 2. Round to the nearest tenth.
 - **a.** What percent of 250 is 138? **b.** 14 is what percent of 15?

OBJECTIVE 2

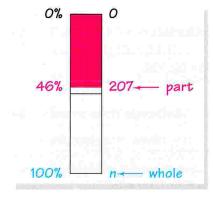
Finding a Whole Amount

Sometimes you know the percent that a part represents, and you want to find the whole amount. For example, your class fundraising committee might announce, "We've collected \$207 so far, which is 46% of our goal!" You can use a proportion to calculate the goal.

3 EXAMPLE

Finding the Whole Amount

207 is 46% of what number?



$$\frac{46}{100} = \frac{207}{n}$$

Write a proportion.

$$46n = 100(207)$$

Write cross products.

$$\frac{46n}{46} = \frac{100(207)}{46}$$

Divide each side by 46.

$$n = 450$$

Simplify.

• 207 is 46% of 450.

Check Understanding Example 3

- 3. Round to the nearest tenth.
 - **a.** 19 is 75% of what number? **b.** 310 is 99% of what number?

EXAMPLE

Real-World Problem Solving

Theaters In 2000, the number of drive-in movie screens in the United States was about 78% of the number in 1990. About how many drive-in screens were there in 1990?

Year	Number of Screens
1990	
1995	847
2000	717

$$\frac{78}{100} = \frac{717}{n}$$
 Write a proportion.
$$78n = 100(717)$$
 Write cross products.
$$\frac{78n}{78} = \frac{100(717)}{78}$$
 Divide each side by 78.
$$n \approx 919$$
 Round to the nearest whole number.

There were about 919 drive-in screens in 1990.

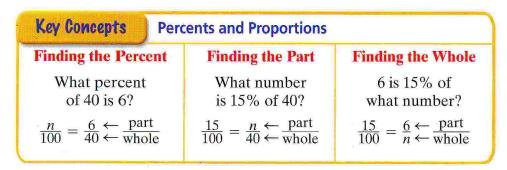
Check Is the answer reasonable? The original problem says that the number of screens in 2000 was 78% of the number in 1990. Check by estimating:

 $78\% \text{ of } 919 \approx 0.8 \times 900 = 720$, which is close to 717, the number for 2000. So the answer is reasonable.

Check Understanding Example 4

4. Refer to the table in Example 4. In 2000, the number of drive-in movie screens was about 20.1% of the number in 1980. Find the number of drive-in screens in 1980.

Here is a summary of how to use proportions to solve percent problems.



Practice and Problem Solving



Practice by Example

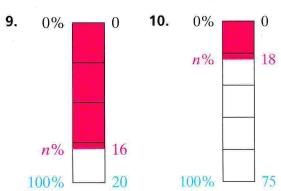
For Exercises 1–22, write and solve a proportion. Where necessary, round to the nearest tenth.

Example 1 (page 315)



- **3.** 80% of 20 is ...
- **4.** 40% of 60 is ...
- **5.** 53% of 70 is ...
- **6.** 18% of 150 is ...
- **7.** 16% of 75 is ■.
- **8.** 92% of 625 is ■.

Example 2 (page 316)



- **11.** \(\bigwedge \) \(\text{of 40 is 30.} \)
- **12.** \(\infty \) of 20 is 4.
- **13.** % of 25 is 13.
- **14.** 75 is \(\infty \) of 250.
- **15.** \(\bigwideta \) \(\text{of 92 is 17.}
- **16.** % of 80 is 14.

Example 3 (page 316)

- **17.** 8 is 25% of ■.
- **18.** 14 is 35% of ...
- **19.** 31 is 49% of ...

- **20.** 45 is 93% of ...
- **21.** 1 is 2% of ...
- **22.** 6 is 98% of ■.

(page 317)

- **Example 4 (23. Population** In 1950, the population of Alaska was about 128,535. That was about 20.5% of the population of Alaska in the year 2000. About how many people lived in Alaska in the year 2000?
 - **24. Banking** At the beginning of the summer Jeri had \$480 in her savings account. That was only 15% of the amount in her savings account at the end of the summer. How much money did Jeri have in her account at the end of the summer?

Apply Your Skills

Write and solve a proportion. Where necessary, round to the nearest whole amount.

- 25. Find 300% of 50.
- **26.** 250% of is 50.
- **27.** Find 60% of 15.
- **28.** 40,571 is \(\bigsim \)% of 76,550.
- **29.** 35% of \blacksquare is 52.5.
- **30.** 121.8 is \(\bigsim \)% of 105.
- 31. Purchasing A bicycle cost \$250 last year. The same bike costs \$200 this year. What percent of last year's cost is this year's cost?

State	Sales Tax
Georgia	7%
Kansas	4.9%
Pennsylvania	6%
South Carolina	5%

Sales Tax The table shows sales tax rates for different states. For each state, find the following amount on a \$15,000 car.

a. the amount of sales tax

b. the car's total cost

33. **Profit** You invested some money and made a profit of \$55. Your profit was 11% of your investment. How much did you invest?

34. Nineteen members, or 38%, of the ski club are going on a ski trip. Find the total number of members in the club.

35. Error Analysis Your class has 26 students, which represents 5% of your school's enrollment. Your friend uses the proportion $\frac{5}{100} = \frac{n}{26}$ to find the number of students in your school. Explain your friend's error.

G Challenge

Write and solve a proportion. Where necessary, round to the nearest tenth.

36. Find $33\frac{1}{3}\%$ of 54.

37. $12\frac{1}{2}\%$ of \blacksquare is 6.

38. \(\mathbb{\text{\tin}\text{\tetx{\text{\te}\text{\text{\text{\text{\text{\texi}\text{\text{\text{\text{\text{\text{\text{\texi}\text{\text{\text{\text{\text{\texi}\text{\texit{\text{\texi}\text{\text{\texi}\tint{\text{\texi}\text{\texi}\text{\text{\text{\text{\text{\texi}\text{\texi}\text{\texit{\t

39. What is $\frac{5}{4}$ % of 145?

40. Open-Ended Write and solve a word problem involving percents.

41. Writing in Math At Pics, all posters are 30% off. At Pacs, all posters are marked $\frac{1}{3}$ off. Which is the greater discount rate? Explain.

42. Reasoning Do a% of b and b% of a represent the same amount? Justify your answer.



Test Prep

Gridded Response

- 43. 42 is 60% of what number?
- **44.** 1.25 is what percent of 25?
- 45. What number is 30% of 75?
- **46.** A student pole-vaulted 5 ft yesterday. Today she vaulted 20% higher. How many feet higher did she vault today?



Mixed Review

Lesson 6-5 Write each number as a percent.

47. 0.08

48. 0.523

49. $\frac{7}{12}$

50. 4.56

Lesson 4-9 Order from least to greatest.

51. 10^3 , 10^{-2} , 10^{-1} , 10^0

52. $2.3 \times 10^4, 2.03 \times 10^5, 2.03 \times 10^4, 2.4 \times 10^3$

Lesson 1-1 **§ 53.** Family Peter has four cousins. Paul has c cousins fewer than Peter. Write an expression for the number of Paul's cousins.

Percents and Equations

What You'll Learn



To write and solve percent equations



To use equations in solving percent problems

... And Why

To solve real-world problems involving earnings and surveys



Check Skills You'll Need

Write each percent as a decimal.

- 1.48%
- 2.5%
- 3.23.8%
- 4. 72.25%
- **5.** 136%
- **6.** 178.5%



New Vocabulary

commission

OBJECTIVE

Writing and Solving Percent Equations

You can solve a percent problem by writing and solving an equation. When you use a percent in an equation, write it as a decimal.

Finding the Part

Key Concepts

Percent Equations

Finding the Percent

What percent of 40 is 6? • 40 = 6

What is 15% of 40?
$$n = 0.15 \cdot 40$$

Finding the Whole

6 is 15% of what?

$$6 = 0.15 \cdot n$$

EXAMPLE

Solving a Percent Equation

What is 85% of 62?

 $n = 0.85 \cdot 62$ Write an equation. Write the percent as a decimal.

$$n = 52.7$$

Simplify.

85% of 62 is 52.7.

Check Understanding Example 1

- 1. Write and solve an equation.
 - **a.** 0.96 is what percent of 10?
- **b.** 19.2 is 32% of what?

You can also write and solve equations having percents greater than 100%.

Reading Math

For help with reading and solving percent equations, see page 324.

EXAMPLE

Percents Greater Than 100%

What percent of 48 is 54?

 $n \cdot 48 = 54$

Write an equation.

 $\frac{48n}{48} = \frac{54}{48}$

Divide each side by 48.

n = 1.125

Simplify.

= 112.5%

Change the decimal to a percent.

54 is 112.5% of 48.

Check Understanding Example 2

- 2. Write and solve an equation.
 - a. What is 145.5% of 20?
- **b.** 380 is 125% of what number?

Interactive lesson includes instant self-check, tutorials, and activities.

OBJECTIVE

Using Equations to Solve Percent Problems

Some sales jobs pay an amount based on how much you sell. This amount is called a **commission**.

3 EXAMPLE

Real-World Problem Solving

Commission A real-estate agent makes a 4.5% commission on property she sells. How much commission does she make on the sale of a house for \$132,500?

words amount of commission is
$$4.5\%$$
 of \$132,500
Let c = amount of commission.
Equation c = $0.045 \cdot 132,500$
 $c = 0.045 \cdot 132,500$
= 5,962.50

The agent's commission is \$5,962.50.

Check Understanding Example 3

3. Royalties A singer receives a 5% royalty on each CD sale. To the nearest cent, find his royalty for a CD that sells for \$16.99.

4 EXAMPLE

Real-World Problem Solving

Surveys The graph shows the results of a survey. There were 1,023 people who answered yes. How many people were surveyed?

Words 1,023 is 93% of number surveyed

Let
$$n = \text{number surveyed}$$
.

Equation 1,023 = 0.93 · n
 $0.93n = 1,023$
 $\frac{0.93n}{0.93} = \frac{1,023}{0.93}$
 $n = 1,100$



Check Understanding Example 4

1,100 people were surveyed.

4. In a survey, 922 people, or about 68.6%, preferred smooth peanut butter to chunky. How many people were surveyed?

Practice and Problem Solving



Practice by Example

Write an equation and solve.

Example 1 (page 320)

- **1.** Find 30% of 30.
- 2. What percent of 40 is 25?

- **3.** 120 is 15% of what number?
- **4.** What percent of 20 is 11?
- **5.** Find 56% of 75.
- **6.** 85% of z is 106,250. What is z?
- **7.** What percent of 25 is 17?
- 8. Find 75% of 840.

Example 2 (page 320)

- **9.** Find 150% of 90.
- **10.** 300% of *a* is 297. What is *a*?
- **11.** What percent of 4 is 9?
- **12.** Find 500% of 12.
- **13.** What percent of 150 is 96?
- **14.** 3.5% of *d* is 0.105. What is *d*?
- **15.** What percent of 1 is 4.7?
- **16.** Find 15% of 150.

(page 321)

- **Example 3 6 17. Royalties** Julius writes novels and receives 12% of the price for each book sold. To the nearest cent, find the royalty Julius receives for a book price of \$7.99.
 - **§ 18. Sports** An agent makes 16% commission on an athlete's signing bonus. If the bonus is \$26,000, what is the agent's commission?

Example 4 (page 321)

For Exercises 19 and 20, the table gives information about videocassette recorders (VCRs) in the United States.

- **19.** The number of households with VCRs in 1995 was about 93% of the number with VCRs in 1998. About how many households had VCRs in 1998?
- **20.** The number of households with VCRs in 1990 was about 87.5% of the number with VCRs in 1993. About how many households had VCRs in 1993?

Households With VCRs

Year	Households (millions)
1980	1
1985	18
1990	63
1995	77
1998	

SOURCE: Statistical Abstract of the United States. Go to www.PHSchool.com for a data update. Web Code: adg-2041

Apply Your Skills

Write and solve an equation. Where necessary, round to the nearest tenth or tenth of a percent.

- **21.** Find 225% of 3.6.
- **22.** What percent of 45 is 24?
- **23.** Find 5.5% of 44.
- **24.** 24% of w is 3.6. What is w?
- **25.** What percent of 8 is 20?
- **26.** 9.2% of *b* is 27.6. What is *b*?
- **27.** 135% of t is 63. What is t?
- **28.** What is 264% of 12?
- **29. Commission** A salesperson receives 5.4% commission. On one sale, she received \$6.48. What was the amount of the sale?
 - **30.** Reasoning Describe a situation in which you would use a percent greater than 100%.

Mental Math Use mental math.

- **31.** What percent of 60 is 30?
- **32.** 100% of t is 100. What **i**s t?
- **33.** Find 5% of 10.
- **34.** What percent of 55 is 11?
- **35.** 50% of *g* is 24. What is *g*?
- **36.** Find 15% of 12.



Writing in Math

Which approach do you

prefer to use in solving

percent problems—the approach you learned in this lesson, or the

one you learned in

Lesson 6-6? Explain.

For Exercises 37 and 38, use the table on page 322. Is each statement true or false? Explain.

- **37.** Reasoning The number of households with VCRs in 1985 was less than 10% of the number of households with VCRs in 1990.
- **38.** The number of households with VCRs in 1985 was more than 1,000% of the number in 1980.
- **39.** Polly got a 20% discount on a computer that regularly cost x dollars. She paid sales tax of 5%. Later she sold the computer for 70% of what she paid for it. Write an expression for the amount Polly received for the computer.



Test Prep

Multiple Choice

- **40.** About what percent of 92 is 63?
 - A. 63%
- **B.** 68%
- C. 92%
- D. 146%



- 41. Seventy students voted for Tim, 25% voted for Li, and the other 40% voted for Mae. How many students voted in the election?
 - F. 200
- **G**. 135
- H. 100
- 1. 70

- 42. What is 158% of 35?
 - **A.** 12.64
- **B.** 20.3
- C. 42.6
- **D.** 55.3

Short Response

- 43. Chan's team won 70% of the 20 games it played. Latisha's team played 15 games and won 80% of them.
 - a. Whose team won the greater number of games?
 - **b.** For part (a), explain your answer.

Mixed Review

Lesson 6-6 Write a proportion. Then solve.

- **44.** \(\bigwidetilde{9}\) of 360 is 45. **45.** 35\(\bigwidetilde{9}\) of 60 is \(\bigwidetilde{9}\).
- **46.** 45 is 1.5% of ...
- Lesson 5-8 47. Creative Writing Ernest started writing a story on a Friday. He worked on the story for $\frac{1}{2}$ h each day. He took 7 h to finish it. On what day did Ernest finish his story?

Lesson 4-7 Simplify each expression.

- **48.** $10^2 \cdot 10^4$
- **49.** $9v^4 \cdot v^5$
- **50.** $(x^3)^7$

Reading Percent Problems

For Use With Page 320, Key Concepts

To go from words to an equation, you first have to recognize a word equation. This requires that you recognize the variable, all operations, and the relationship. Percent problems can have various word equations, but each is equivalent to one of the three types in this table.

Key Concepts

Percent Equations

Finding the Percent

What percent of 40 is 6?

$$n \cdot 40 = 6$$

Finding the Part

What is 15% of 40?

$$n = 0.15 \cdot 40$$

Finding the Whole

6 is 15% of what?

$$6 = 0.15 \cdot n$$

EXAMPLE

Six of 40 students wear at least one ring. What percent is this?

What percent of 40 is 6? Recognize the percent equation.

of 40 is 6

What percent is what you must find. Let a variable, n, represent this value.

n

• 40 is 6

The word of suggests multiplication.

 $\cdot 40 = 6$ The word is suggests equality.

Now you have a math equation that you can solve.

 $\frac{n \cdot 40}{40} = \frac{6}{40}$

Divide each side by 40.

n = 0.15 Simplify.

n = 15% Write as a percent.

EXERCISES

Restate, if necessary, the question to match a type of question in the table. Then solve.

1. Seven is what percent of 21?

- **2.** Eighteen is thirty percent of what number?
- **3.** How much is twenty percent of 50?
- **4.** What percent of 6 is 40?
- **5.** Sixty runners, or 80% of all entries, finished the course. How many entries were there?
- 6. Thirty students are in Ms. Payne's history class. Ninety percent of the students brought signed permission forms for a field trip. How many students brought permission forms?

Percent of Change

OBJECTIVE

Finding Percent of Increase



Exploring Percent of Change

- 1. Find the change in population from 1980 to 1990 for each state.
- 2. Which state had the greater change in population?
- **Populations of Two States**

State	1980	1990
California	23,668,000	29,786,000
Nevada	800,000	1,202,000

- 3. Write the ratio $\frac{\text{change in population}}{1980 \text{ population}}$ for each state. Then write each ratio as a percent.
- 4. Compare the two percents. Which state had the greater population change in terms of percent?

The percent a quantity increases or decreases from its original amount is the **percent of change**.

percent of change =
$$\frac{\text{amount of change}}{\text{original amount}}$$

EXAMPLE

Finding Percent of Increase

Find the percent of increase from 4 to 7.5.

amount of increase =
$$7.5 - 4 = 3.5$$

percent of increase = $\frac{\text{amount of increase}}{\text{original amount}}$
= $\frac{3.5}{4}$
= $0.875 = 87.5\%$

The percent of increase from 4 to 7.5 is 87.5%.

Check Understanding Example 1

- 1. Find each percent of increase.
 - **a.** from 100 to 114 **b.** from 2.0 to 3.2
- **c.** from 4,000 to 8,500

What You'll Learn



To find percent of increase



To find percent of decrease

... And Why

To solve real-world problems involving environmental management

Check Skills You'll Need

Write each decimal as a percent.

1. 0.46 2, 2,47

3. 0.03 4. 5.236

Ker help, go to Lesson 6-5.

New Vocabulary

percent of change

Interactive lesson includes instant self-check, tutorials, and activities.

EXAMPLE

Real-World Problem Solving

Waste Management

The annual production of municipal solid waste in the United States has more than doubled since 1960. Find the percent of increase from 1960 to 1990.

amount of increase
$$= 205 - 88 = 117$$

percent of increase

$$=\frac{\text{amount of increase}}{\text{original amount}}$$

$$=\frac{117}{88}$$

$$= 1.329\overline{54} \approx 133\%$$



Source: Environmental Protection Agency. Go to www.PHSchool.com for a data update. Web Code: adg-2041

The percent of increase from 1960 to 1990 was about 133%.

Check Understanding Example 2

2. Waste Management Find the percent of increase in solid-waste production from 1970 to 1980. Round to the nearest percent.

OBJECTIVE

Finding Percent of Decrease

You also can find percent of decrease.

EXAMPLE

Finding Percent of Decrease

Find the percent of decrease from 1,500 to 1,416.

amount of decrease =
$$1,500 - 1,416 = 84$$

percent of decrease
$$=$$
 $\frac{\text{amount of decrease}}{\text{original amount}}$

$$=\frac{84}{1,500}$$

$$= 0.056 = 5.6\%$$

The percent of decrease is 5.6%.

Check Understanding Example 3

- 3. Find each percent of decrease. Where necessary, round to the nearest tenth of a percent.
 - **a.** from 9.6 to 4.8
- **b.** from 202 to 192
- **c.** from 854.5 to 60.6

EXERCISES

Practice and Problem Solving



Find each percent of increase.

Examples 1 and 2 (pages 325, 326)

- **1.** from 30 to 39 **2.** from 50 to 66
- **4.** from 48 to 60 **5.** from 32 to 76
 - **6.** from 32 to 76 **6.** from 5 to 5.5
- **7.** from 55 to 176
- 8. from 38 to 95
- 9. from 2.5 to 3

3. from 4 to 4.5

■ 10. Life Spans In the United States in the 20th century, average life expectancy increased from about 47 years to about 77 years. Find the percent of increase to the nearest percent.

Example 3 (page 326)

Find each percent of decrease. Where necessary, round to the nearest tenth of a percent.

- **11.** from 60 to 48
- **12.** from 180 to 54
- **13.** from 180 to 108

- **14.** from 280 to 126 **1**!
 - **15.** from 240 to 90
- **16.** from 42 to 35

- **17.** from 64 to 24
- **18.** from 6.5 to 4.8
- 19. from 7.4 to 2.4
- 20. A computer that cost \$1,099 last year costs \$999 this year.
- 21. A racing bicycle that cost \$1,500 new costs \$845 used.
- **B** Apply Your Skills

Find each percent of change. Tell whether the change is an increase or a decrease. Where necessary, round to the nearest tenth of a percent.

- **22.** from 96 to 78
- 23. from 90 to 75
- 24. from 80 to 95

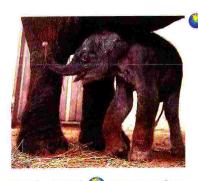
- **25.** from 45 to 105
- **26.** from 27 to 72
- **27.** from 120 to 95

- **28.** from 87 to 108
- **29.** from 59 to 127
- **30.** from 77 to 13
- **31. Error Analysis** Eva's first step in finding the percent of change from 7 to 8 was to write $\frac{8-7}{8} = \frac{1}{8}$. Explain Eva's error.
- **32. Economics** The average cost of a gallon of gasoline was \$1.29 in 1997 and \$1.12 in 1998. Find the percent of decrease.

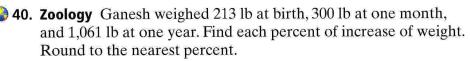
Mental Math Use mental math to find each percent of change. Tell whether the change is an increase or a decrease.

- **33.** from 25 to 30
- **34.** from 40 to 45
- **35.** from 50 to 45
- **36.** from 100 to 101.1 **37.** from 40 to 20
- **38.** from 15 to 12

- **G** Challenge
- **39.** The population of Growtown increased from 10,000 to 13,000 in one year. In the same year, the population of Slowtown decreased from 30,000 to 24,000.
 - a. Find each town's percent of increase or decrease in population.
 - **b.** If each town maintains the same rate of change, within how many years will the population of Growtown exceed that of Slowtown?



Real-World Real-World Ganesh was the first elephant born at the Cincinnati Zoo.



- a. from birth to one month
- **b.** from one month to one year
- c. from birth to one year
- d. Writing in Math Explain why the sum of the percent increase from birth to one month and from one month to one year does not equal the percent increase from birth to one year.
- **41. a. Reasoning** 100 is increased by 10%. The result is decreased by 10%. Is the final result 100? Explain.
 - **b.** Compare the final result in part (a) to 100, the original number. Find the percent of change.



Test Prep

Multiple Choice

- 42. The price of an item is \$5.99. With sales tax, you pay \$6.35. About what percent of the price of the item is the sales tax?
 - A. 3%
- B. 4%
- C. 5%
- 43. Kayla was 36 in. tall at age 3. Today, at age 5, she is 42 in. tall. To the nearest percent, what is the percent of change in Kayla's height?
 - F. 2%
- G. 8%
- H. 14%
- I. 17%



- 44. A share of stock sold for \$32.13 yesterday. Today, it is selling for \$30.08. What is the approximate percent decline in the stock price?
 - A. 5%
- B. 6%
- C. 7%
- D. 8%
- 45. What is the percent of change from 148 to 37?
 - F. 3%
- **G**. 25%
- H. 75%
- I. 111%

Extended Response

46. 200 is decreased by 5%. The result is increased by 5%. What is the final result? Explain why the result is less than 200. Show your work.

Mixed Review

- Lesson 6-7 @ 47. Astronomy The Space Surveillance Center in Colorado tracks about 8,500 objects in orbit around Earth. All but about 500 objects are junk from past space missions. What percent are junk? Round to the nearest percent.
- Lesson 5-3 Find each sum or difference.

48.
$$5\frac{3}{4} - 2\frac{5}{8}$$

49.
$$-4\frac{1}{3} + 2\frac{1}{2}$$
 50. $-6\frac{1}{3} - 6\frac{1}{3}$

50.
$$-6\frac{1}{3} - 6\frac{1}{3}$$

Evaluate each expression. Lesson 4-2

51.
$$3x^2$$
 for $x = -5$

52.
$$[(3 + 12)4]^2$$

51.
$$3x^2$$
 for $x = -5$ **52.** $[(3 + 12)4]^2$ **53.** $(7 + 4y)^2$ for $y = -2$

Markup and Discount

OBJECTIVE

Finding Markups

To make a profit, stores charge more for merchandise than they pay for it. The amount of increase is called the markup. The percent of increase is the percent of markup.

EXAMPLE

Real-World Problem Solving

Music Sales A music store's percent of markup is 67%. A CD costs the store \$10.15. Find the markup.

 $markup = percent of markup \cdot store's cost$

 $= 0.67 \cdot 10.15$

 ≈ 6.80

Simplify. Round to the nearest cent.

The markup is \$6.80.

Check Understanding Example 1

1. A clothing store pays \$56 for a jacket. The store's percent of markup is 75%. Find the markup for the jacket.

The store's cost plus the markup equals the selling price.

EXAMPLE

Real-World Problem Solving

Retailing A computer store pays \$6 for a computer mouse. The percent of markup is 75%. Find the mouse's selling price.

 $0.75 \cdot 6 = 4.50$

Multiply to find the markup.

6.00 + 4.50 = 10.50 Cost + markup = selling price.

The selling price is \$10.50.

Check Understanding Example 2

2. A \$5 cap has a 70% markup. Find the selling price.

OBJECTIVE

Finding Discounts

When an item goes on sale, the amount of the price decrease is the **discount.** The percent of decrease is the percent of discount. Sale price = regular price - discount.

What You'll Learn



To find markups



To find discounts

... And Why

To solve real-world problems involving price markups and discounts

Check Skills You'll Need

Write an equation and solve. Round to hundredths as needed.

- 1. What is 75% of \$82?
- **2.** What is 42% of \$170?
- **3.** What is 5.5% of \$24?
- **4.** What is 80% of \$15.99?
- For help, go to Lesson 6-7.

New Vocabulary

- markup
- discount

Interactive lesson includes instant self-check, tutorials, and activities.



20% off means a discount of 20%.

EXAMPLE

Finding Discount

Recreation Athletic shoes that regularly sell for \$85.99 are on sale for 20% off. Find the discount.

 $discount = percent of discount \cdot regular price$

 $= 0.20 \cdot 85.99$

 ≈ 17.20

Simplify. Round to the nearest cent.

The discount is \$17.20.

Check Understanding Example 3

3. Pants priced at \$21.99 are marked 15% off. Find the discount.

Here are two ways to use percent of discount to find a sale price.

More Than One Way

A video game that regularly sells for \$39.95 is on sale for 20% off. What is the sale price?

Eric's Method

Find the discount. Then find the sale price.

discount = percent of discount · regular price

 $= 0.20 \cdot 39.95$

= 7.99

sale price = regular price - discount

= 39.95 - 7.99

= 31.96

The sale price is \$31.96.



Find the sale price directly. The sale price equals 100% of the regular price minus 20% of the regular price.

sale price = $(100\% - 20\%) \cdot \text{regular price}$

= 80% · regular price

= 0.80(39.95)

= 31.96

The sale price is \$31.96.

Choose a Method

- 1. Which method do you prefer? Explain.
- 2. Find the sale price if the percent of discount is 25%. Round to the nearest cent.



EXERCISES

Practice and Problem Solving



For Exercises 1–5, find each markup.

Example 1 (page 329)

- 1. cost: \$1.50 percent of markup: 70%
- **2.** cost: \$38 percent of markup: 58%
- **3.** cost: \$111.00 percent of markup: 50%
- 4. cost: \$18 percent of markup: 35%
- **S. Beach Gear** A beach store pays \$11.40 for each beach umbrella. The store's percent of markup is 75%.

Example 2 (page 329)

For Exercises 6–10, find each selling price.

- 6. cost: \$6 percent of markup: 75%
- 7. cost: \$2.66 percent of markup: 50%
- 8. cost: \$149.99 percent of markup: 100%
- **9.** cost: \$67.20 percent of markup: 10%
- 10. Clothing A clothing store pays \$15 for a shirt. The percent of markup is 85%.

Example 3 (page 330)

For Exercises 11–15, find each discount and sale price.

- **11.** regular price: \$100 percent of discount: 27%
- **12.** regular price: \$24.50 percent of discount: 20%
- **13.** regular price: \$700 percent of discount: 30%
- **14.** regular price: \$8.49 percent of discount: 5%
- 😂 15. Footwear Boots, regularly \$125, are on sale for 30% off.

Apply Your Skills

Find each selling price. Where necessary, round to the nearest cent.

- **16.** regular price: \$180 percent of discount: 40%
- **17.** regular price: \$14.99 percent of discount: 15%
- **18.** cost: \$9.99 percent of markup: 60%
- **19.** cost: \$15 percent of markup: 15%
- **20. Shopping** An \$11 shirt is on sale for 10% off.
 - a. Describe two different methods of finding the sale price.
 - **b.** Use one of the methods to find the sale price.
- **21. Video Sales** Store A is selling a video for 20% off the store's regular price of \$25.95. Store B is selling the same video for 30% off the store's regular price of \$29.50. Which store's sale price is lower? How much lower is it?
 - 22. Writing in Math Identical sweaters are on sale in two different stores. The sale price in Store A is 30% off the regular price of \$25. The sale price in Store B is 40% off the regular price of \$30. Which sweater is the better buy? Explain.



- 23. Estimation Suppose you want to buy three books that cost \$6.95, \$9.95, and \$10.95. The bookstore is having a $\frac{1}{4}$ -off sale. Your state charges sales tax of 5% of an item's final price. You have \$20. Do you have enough money? Justify your answer.
- **24.** Reasoning A store buys an item for x and sells it for y. Write expressions for the markup and the percent of markup.
- 25. Shoes cost a store \$56.40. The markup is 17%. Find the selling price by two different methods.



Test Prep

Multiple Choice

- Take It to the **NET** Online lesson quiz at www.PHSchool.com ··· Web Code: ada-0609
- 26. A \$59.50 coat is on sale for \$36.50. About how much is the percent of discount?
 - A. 37%
- B. 39%
- C. 61%
- D. 63%
- 27. Michael buys a new lawn mower with a 20% discount coupon. The regular price is \$359.99. How much is the discounted price?
 - F. \$20.00
- **G**. \$72.00
- H. \$287.99
- I. \$339.99

Short Response

28. Store A advertises a printer at "15% off its regular price of \$79.95." Store B next door advertises the same printer at "20% off of \$83.95." (a) Which printer would you buy? (b) Explain your answer.

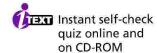
Mixed Review

- Lesson 6-8 Find each percent of decrease. Round to the nearest tenth.
 - **29.** from 90 to 70
- **30.** from 44.4 to 14.8 **31.** from 1,750 to 1,125
- Lesson 5-3 3 32. Recipes A bread recipe calls for $6\frac{1}{2}$ cups of flour. You have $4\frac{3}{4}$ cups. How much more flour do you need?
- Draw a coordinate plane. Graph each point. Lesson 1-10
 - **33.** A(1,0)
- **34.** B(-2,3)
- **35.** C(-1,2)



Checkpoint Quiz 2

Lessons 6-5 through 6-9



Compare. Use >, <, or = to complete each statement.

1.
$$\frac{14}{25}$$
 \square 56%

3.
$$\frac{3}{11} = 27\%$$

Write and solve an equation.

- **4.** Find 33% of 120.
- **5.** Find 125% of 42.
- **6.** What percent of 5.6 is 1.4?
- **7.** 15% of *q* is 9.75. What is *q*?
- 8. A car originally priced at \$12,000 is sold at a 20% discount. Find the sale price.



Making a Table

For Use With Lesson 6-10

You can use the **TABLE** feature of a graphing calculator to show values that result from repeated operations.

EXAMPLE

The population of a town increases at the rate of 0.5% each year. Today the town's population is about 5,000. About what will the population be next year? in 5 years?

TABLE lets you show the first two columns of the following table for population P = 5,000. In the table, note that after each year, the population is 1.005 times the population at the start of the year.

Year	Population Start of Year	Population End of Year
0	P	P(1.005)
1	P(1.005)	$P(1.005)^2$
2	$P(1.005)^2$	$P(1.005)^3$
3	$P(1.005)^3$	$P(1.005)^4$
4	$P(1.005)^4$	$P(1.005)^5$
• • •		

Press $Y_1 = 5000(1.005)^X$.

In **TBLSET,** set TblStart = 0 and Δ Tbl = 1.

Press **TABLE**, to view the first two columns of the table.

The table shows that after 1 year, the population will be 5,025. After 5 years, the population will be 5,126.

1	Υ	Y	1	I	Ĭ				1						1			
0	5555555	5(0)!)	57	5(7	(5		Z	1							
111	5555	5	111) (() !!	025)(26	100	5051	Control of the last of the las	2	133							

You can use a table like the one above as you work the Lesson 6-10 Example. Be sure to compare your table with the table on page 335.

EXERCISES

- 1. Find the population of the town above after 10 years.
- 2. In what year will the population exceed 5,500?
- **3.** Suppose the growth rate of the town is 0.6%. What will its population be at the end of 1 year? 5 years? 10 years? In what year will its population exceed 5,500?
- **4.** A nearby town has population 6,000 and a growth rate of 0.5%. What will its population be at the ends of 1, 5, and 10 years?

6-10 Problem Solving

Make a Table

What You'll Learn



To solve problems by making a table

... And Why

To solve real-world problems involving population estimates

Check Skills You'll Need Solve.

1. For two weeks, you double the amount of money you save each day. You save \$.01 the first day. How much money will you have at the end of the two weeks?

For help, go to Lesson 1-8.

OBJECTIVE



Make a Table

Math Strategies in Action

Have you ever watched a baseball game at a field that doesn't have a scoreboard? It's hard to keep track of the score!

A scoreboard is a type of table. You can use tables to organize information. Tables are particularly helpful in solving problems that require several steps.



1 EXAMPLE

Real-World Problem Solving

Population Growth At the beginning of the year 2000, the population of the United States was about 273.5 million. The rate of population growth was about 0.85% per year. If that rate continues, what will the population be at the beginning of 2010?

Read and Understand

Read the problem carefully.

- 1. What information are you asked to find?
- 2. What information will you need to use to solve the problem?

Plan and Solve

Decide on a strategy. You can use the percent of increase to predict the population increase for each year from 2000 to 2010. You can make a table to organize your predictions for each year.

- **3.** How can you find the increase in population from the beginning of 2000 to the end of that year?
- **4.** How can you find the population at the beginning of 2001?

5. The percent of increase is the same each year. Does that mean that the increase in population also will be the same each year? Explain your reasoning.

Copy and complete the table below.

- **6.** Find the numbers for Column 4 by multiplying the numbers in Columns 2 and 3. Round to the nearest tenth of a million.
- 7. Find the numbers for Column 5 (and, hence, the next Column 2 entries) by adding the numbers in Columns 2 and 4.

1	2	3	4	5
Year	Population at Beginning of Year (millions)	Rate of Increase (0.85%)	Increase in Population (millions)	Population at End of Year (millions)
2000	273.5	0.0085	2.3	275.8
2001	275.8	0.0085	2.3	278.1
2002	278.1	0.0085	2.4	280.5
2003	280.5	0.0085	2.4	282.9
2004	282.9	0.0085	2.4	285.3
2005	285.3	0.0085	2.4	287.7
2006	287.7	0.0085	2.4	290.1
2007	290.1	0.0085		
2008	100	0.0085		
2009		0.0085	and the second s	
2010			d.	1

- **8.** Sometimes the number in Column 4 changes from one year to the next, and sometimes it does not change. Explain.
- **9.** What is your prediction for population at the beginning of 2010?

Look Back and Check

10. Your friend says that she knows a quicker way to find the answer. Simply multiply $273.5 \cdot 0.0085 \cdot 10$ to find the increase for the ten-year period 2000 to 2010. Do you agree with your friend's approach? Explain your reasoning.

✓ Check Understanding

11. Suppose the annual percent of increase in population is 0.9%. At that rate, what will the population be at the beginning of 2010?

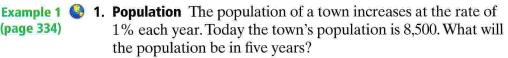
Practice and Problem Solving



Practice by Example

Make a table to solve each problem.

(page 334)





2. Biology A microbe population increases 100% every 10 min. If you start with 1 microbe, how many will you have at the end of 1 h?

3. Cher has forgotten the combination to her locker. She knows it consists of four numbers—3, 5, 7, and 9—but she can't recall the order. She decides to try every possible order until she gets the right one. How many possible orders are there?

Microbe cultures grow on agar gel in petri dishes.

Real-World Connection

Banking At the beginning of the year 2000, Bob put \$100 in a savings account. The bank pays Bob 5% interest on his total savings at the end of each year including all interest added to the account. Assume the interest rate continues and Bob does not deposit any additional money in the account. How much will he have in his savings account, to the nearest cent, after 5 interest payments?



Apply Your Skills

Use any strategy to solve each problem.

Strategies

- 5. Paco has four pairs of jeans and four T-shirts. How many outfits of a T-shirt and a pair of jeans can Paco make?
- Account for All **Possibilities**
- **6. Geometry** The length of a rectangle is twice the width. The perimeter of the rectangle is 42 cm. Find the length and width.
- Draw a Diagram
- Look for a Pattern
- 7. Number Sense The difference of two numbers is 18. The sum of the two numbers is 34. What are the two numbers?
- Make a Model Make a Table
- § 8. Capacity You fill a container $\frac{3}{4}$ full of water. The amount of water now in the container is 6 quarts. How much can the
- Simplify the Problem Simulate the Problem
- container hold?
- Solve by Graphing
- **9. Ticket Sales** A family went to the movies. Tickets cost \$4 for each child and \$6 for each adult. The total admission charge for the
- Try, Test, Revise
- family was \$26. List all the possible numbers of adults and children in the family.
- Use Multiple Strategies
- **10.** Number Sense A number *n* is multiplied by $\frac{5}{8}$. The product is subtracted from $\frac{2}{3}$. The result is $\frac{7}{12}$. What is n?
- Work Backward Write an Equation Write a Proportion
- 11. Geometry The height of a triangle is half the length of its base. The area of the triangle is 12.25 cm². Find the height.



12. Water Resources Water for irrigation is measured in *acre-feet*. One acre-foot is the volume of water that would cover one acre of land to a depth of one foot. How many acre-feet of water would it take to cover 600 acres to a depth of one inch?



Test Prep

Multiple Choice

- 13. What is the percent of increase from 1.32 to 1.44, rounded to the nearest percent?
 - A. 8%
- B. 9%
- C. 12%
- D. 15%



- 14. A store pays \$50.00 for a coat. Its markup is 25%. Later, it puts the coat on sale at 20% off. What is the sale price of the coat?
 - F. \$40.00
- **G.** \$50.00
- H. \$52.50
- I. \$62.50
- 15. The width of a rectangle is half the length. The perimeter of the rectangle is 54 in. What is the length of the rectangle?
 - **A.** 6 in.
- **B.** 9 in.
- C. 18 in.
- **D.** 36 in.

Short Response

- **16.** You must give a customer \$.40 in change.
 - a. In how many ways can you do this without using pennies? **b.** For part (a), make a table.

Mixed Review

Lesson 6-9 Find each sale price.

- **17.** regular price: \$39 percent of discount: 30%
- **18.** regular price: \$159.95 percent of discount: 20%

Lesson 6-4

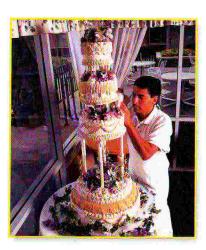
19. Probability What is the probability that a digit selected at random from the number 364,892 is a multiple of 3?

Lesson 4-4 Write in simplest form.

- **20.** $\frac{16}{36}$

- **21.** $\frac{10x}{65x}$ **22.** $\frac{8ab}{2bc}$ **23.** $\frac{12x^2y}{9xy^2}$

Math at Work



Caterers provide food for parties, weddings, and other events. They plan the menu, buy the ingredients, cook the food, and provide the waitstaff. Some will even arrange for music and seating. For each event, a caterer determines the cost per guest.

The catering business requires a thorough knowledge of ratios, proportions, and percents.





Using Estimation

Estimation may help you find an answer, check an answer, or eliminate answer choices. If, however, an incorrect choice is very close to the correct choice, you will still have to find the exact answer.

EXAMPLE

A coat regularly priced at \$89.95 is on sale at 40% off. What is the sale price of the coat?

A. \$35.98

B. \$49.95

C. \$53.97

D. \$54.03

\$89.95 ≈ \$90

40% is less than half. The discount will be less than \$45, which is half of \$90. So, the sale price will be more than \$45.

This eliminates choice A.

 $40\% \text{ of } 90 = 0.4 \cdot 90$

= 36

90 - 36 = 54, so the sale price is about \$54. This eliminates choice B.

Choices C and D are both very close to \$54, so you have to compute to find the answer is choice C.

EXERCISES

In Exercises 1–3, which answer choices can you eliminate by using estimation? Explain.

1. A store pays \$8.50 for a case of scented candles. The store's percent of markup is 110%. What price will the store charge for the case of candles?

A. \$9.35

B. \$16.15

c. \$17.85

D. \$18.15

2. A store is having a sale. Laverne looks at a \$79.80 necklace that is on sale for \$59.85. What is the percent of discount?

F. 20%

G. 25%

H. $33\frac{1}{3}\%$

I. 40%

3. An entomologist noted that in the past five years in the local forest preserve, the population of bees decreased by 35%. In her study, she recorded that the number of active hives five years ago was 35. If the number of active hives decreased at the same rate as the bee population, how many hives are there today?

A. 17 hives

B. 19 hives

C. 21 hives

D. 23 hives

4. For Exercise 3, explain how you can use estimation in two different ways to help you with the answer choices.



Chapter Review

Vocabulary

certain event (p. 306) commission (p. 321) complement (p. 306) cross products (p. 294) discount (p. 329) event (p. 305) impossible event (p. 306) indirect measurement (p. 300) markup (p. 329) odds (p. 307) outcomes (p. 305) percent (p. 310) percent of change (p. 325) probability (p. 305)

proportion (p. 294) rate (p. 289) ratio (p. 288) scale drawing (p. 300) similar figures (p. 299) unit rate (p. 289)



Reading Math Understanding Vocabulary

Match the vocabulary terms with their descriptions.

- 1. a comparison of two quantities by division
- 2. a ratio that compares a number to 100
- 3. the amount charged for an item above the cost
- **4.** the amount of a price decrease
- 5. a ratio that compares quantities in different units
- **6.** a rate that has a denominator of 1
- 7. two equal ratios

- a. markup
- **b.** percent
- c. ratio
- d. discount
- e. unit rate
- f. proportion
- q. rate



Skills and Concepts

6-1 Objectives

- To write and simplify ratios (p. 288)
- To find rates and unit rates (p. 289)

A ratio is a comparison of two quantities by division. A rate is a ratio that compares quantities in different units. A unit rate is a rate that has a denominator of 1.

Write each ratio as a fraction in simplest form.

- **8.** 9:24
- **9.** 20 : 35
- **10.** 15 : 20
- **11.** 100 : 130

Write each ratio as a unit rate.

- **12.** 150 mi in 3 h
- **13.** \$9.45 for 5 lb
- **14.** 270 words in 3 min

6-2 Objectives

- To solve proportions (p. 294)
- To use proportions to solve problems (p. 295)

A proportion is an equality of ratios. To solve a proportion, write the cross products, and then solve.

Solve. Round to the nearest tenth where necessary.

- **15.** $\frac{5}{6} = \frac{n}{42}$

- **16.** $\frac{53}{2} = \frac{18}{r}$ **17.** $\frac{15}{a} = \frac{30}{98}$ **18.** $\frac{m}{150} = \frac{21}{25}$

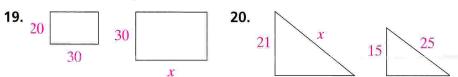
6-3 Objectives

- ▼ To solve problems that involve similar figures (p. 299)
- ▼ To solve problems that involve scale drawings (p. 300)

Similar figures have the same shape, but not necessarily the same size. In similar figures, the corresponding angles have equal measures and the corresponding sides are proportional.

A scale drawing is an enlarged or reduced drawing of an object.

The figures in each pair are similar. Find x.



21. A map has a scale of 1 cm : 75 km. What is the distance on the map for an actual distance of 37.5 km?

6-4 Objectives

- To find probability (p. 305)
- To find odds (p. 307)

Outcomes are the possible results of an action. An **event** is any outcome or group of outcomes. When outcomes are equally likely, you can use formulas to find the **probability** of an event and the **odds** in favor of an event.

probability =
$$\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

odds = $\frac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}}$

Suppose you select a letter at random from the letters in the word EXPONENT. Find the probability of each event. Then find the odds in favor of the event.

22. selecting P

23. selecting N

24. selecting a vowel

6-5 Objectives

- To write percents as fractions and decimals (p. 310)
- To write decimals and fractions as percents (p. 311)

A **percent** is a ratio that compares a number to 100.

Write each percent as a fraction in simplest form and as a decimal.

25. 24%

26. 72%

27. 8%

28. 0.5%

Write each number as a percent. Round to the nearest tenth of a percent.

29. 0.3

30. 0.33

31. 🛓

32. 0.35

33. $\frac{16}{18}$

34. 0.021

35. $\frac{120}{50}$

36. 0.0064

6-6 and 6-7 Objectives

- ▼ To find a part of a whole and a percent (p. 315)
- To find a whole amount (p. 316)
- ▼ To write and solve percent equations (p. 320)
- ▼ To use equations in solving percent problems (p. 321)

Solve percent problems by using a proportion or an equation.

Write and solve a proportion.

- **37.** Find 15% of 48.
- **38.** 20% of *x* is 30. What is *x*?
- **39.** What percent of 300 is 90?
- **40.** 125% of *y* is 100. What is *y*?

Write and solve an equation.

- **41.** 35% of *a* is 70. What is *a*?
- **42.** Find 68% of 300.
- **43.** What percent of 180 is 9?
- **44.** What percent of 56 is 3.5?

6-8 and 6-9 Objectives

- ▼ To find percent of increase (p. 325)
- ▼ To find percent of decrease (p. 326)
- ▼ To find find markups (p. 329)
- ▼ To find discounts (p. 329)

A **percent of change** is the percent by which a quantity increases or decreases from its original amount.

percent of change =
$$\frac{\text{amount of change}}{\text{original amount}}$$

Markup is a real-world application of percent of increase. **Discount** is a real-world application of percent of decrease.

Find each percent of change. Tell whether the change is an increase or a decrease.

- **45**. 120 to 90
- **46.** 148 to 37
- **47.** 285 to 342
- **48.** 1,000 to 250
- **49.** A cap that cost a retailer \$5 was marked up by 75%. Find the selling price.
- **50.** Peaches that are usually priced at \$2/lb are on sale for 15% off. Find the sale price.

6-10 Objectives

To solve problems by making a table (p. 334) Make a table to organize information or to solve problems that have several steps.

- **51.** Alicia bikes 25% of a 100-mi trip on the first day. She bikes $\frac{1}{3}$ of the remaining distance on the second day. On the third day, she bikes 40% of the remaining distance. Make a table to find the number of miles left in Alicia's trip.
- **52.** Describe how you could use a table together with another problem-solving strategy that you have studied. Justify your answer with an example.



Chapter Test

Find each unit rate.

- 1. A car travels 84 mi on 3 gal of gas.
- 2. A car travels 220 mi in 4 h.

Write = or \neq to complete each statement.

3.
$$\frac{7}{8} = \frac{40}{42}$$

4.
$$\frac{3}{5} = \frac{45}{75}$$

5.
$$\frac{12}{18} = \frac{18}{12}$$

6.
$$\frac{5}{9} = \frac{25}{81}$$

Solve each proportion.

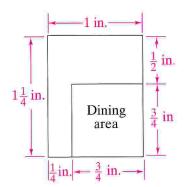
7.
$$\frac{x}{8} = \frac{90}{120}$$

8.
$$\frac{0.8}{90} = \frac{5.6}{y}$$

Write a proportion to describe each situation. Then solve.

- **9.** Three cans of dog food sell for 99¢. Find the cost of 15 cans.
- **10.** A photo that measures 5 in. by 7 in. is enlarged to 7.5 in. by *b* in.
- **11.** A student reads 45 pages in 2 h and *x* pages in 3 h.

For Exercises 12–14, use the drawing below. The length of the kitchen in the drawing is $1\frac{1}{4}$ in. The actual length is 20 ft.



- **12.** What is the scale of the drawing?
- 13. What is the actual width of the kitchen?
- **14.** What are the actual length and width of the dining area?

Find each probability for one roll of a number cube.

15. *P*(1)

16. *P*(1 or 2)

17. *P*(not 2 or 6)

18. P(greater than 1)

Write each decimal as a percent.

19. 0.37

20. 0.005

21. 1.02

Write each fraction as a percent.

22. $\frac{5}{8}$

23. $\frac{7}{16}$

24. $\frac{5}{4}$

Solve.

- **25.** What percent of 400 is 20?
- 26. Find 45% of 12.
- **27.** 20% of *c* is 24. What is *c*?
- 28. What percent of 3 is 15?
- 29. Find 125% of 50.
- **30.** 60% of y is 75. What is y?

Find each percent of change. Tell whether the change is an increase or a decrease. Round to the nearest tenth of a percent.

31. from 60 to 36

32. from 18 to 24

33. from 15 to 25

34. from 85 to 50

35. from 8.8 to 30

36. from 1.2 to 0.2

- **37.** A salesperson made a \$128 commission selling merchandise. His commission rate was 5%. Find the dollar amount of his sales.
- **38.** A bicycle that usually sells for \$230 is on sale for 15% off. Find the sale price.
- **39.** Writing in Math Explain the difference between a markup and a discount.
- **40.** In how many ways can you make \$.35 in change without using pennies?



Test Prep

Multiple Choice

1. What is 56,500,000,000 written in scientific notation?

A. $5.65 \cdot 10^8$

B. $56.5 \cdot 10^{10}$

C. $5.65 \cdot 10^{10}$

D. $565 \cdot 10^9$

2. What is x when $\frac{2}{3}x = 2\frac{2}{9}$?

F. $\frac{3}{10}$ G. $2\frac{5}{8}$ H. $3\frac{1}{3}$

I. 7€

3. What is the correct symbol?

D. ≤

4. What is the unit rate for a ball moving 252 ft in 4 s?

F. 252:4

G. 63 ft/s

H. 252 ft/s

1,008 ft/s

5. What percent of 63 is 41?

A. about 82%

B. about 65%

C. about 41%

D. about 0.65%

6. Which equation has −4 as a solution?

F. 9z = 36

G. $-\frac{36}{z} = -9$

H. z + 9 = 5

1. z-9=-5

7. Which has the lowest unit price?

A. 10 oz for \$.30

B. \$.56 for 20 oz

C. 30 oz for \$.87

D. \$1.16 for 40 oz

Gridded Response

- **8.** Evaluate 6x 9 for x = 11.
- 9. Simplify $(-1)^8 \cdot (-2)^0$.
- **10.** Solve y + 0.5 = 3.
- 11. Find 40% of 40.

Write each fraction as a decimal.

12. $\frac{42}{50}$

13. $\frac{16}{20}$ 14. $\frac{33}{25}$

15. $\frac{6}{80}$

16. $\frac{524}{200}$ **17.** $\frac{45}{1.000}$

Find each probability for one roll of a number cube.

18. P(5 or 6)

19. *P*(less than 4)

The scale on a map is 1 in. = 5 mi. Find the actual distance in miles for each map distance.

20. 5.5 in.

21. 12 in.

22. 9.75 in.

Solve each proportion.

23. $\frac{5}{8} = \frac{15}{n}$

24. $\frac{28}{8} = \frac{14}{25}$

25. $\frac{n}{9} = \frac{40}{12}$

26. $\frac{6}{21} = \frac{s}{70}$

Write each ratio as a fraction in simplest form.

27. 20:45

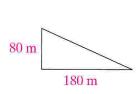
28. 8:96

29. 30:36

30. 120:80

Short Response

31. For the similar figures, (a) write a proportion to solve for x. (b) Find x.





In Exercises 32 and 33, (a) find the sale price. Round to the nearest cent where necessary. (b) Explain your answer.

- 32. regular price: \$58 percent of discount: 45%
- 33. regular price: \$15.98 percent of discount: 80%

Extended Response

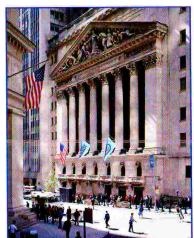
34. Which is the better buy: (a) Brand A with a regular price of \$15.98 and a 30% discount, or Brand B with a regular price of \$18.50 and a 40% discount? (b) How much do you save with the better buy? (c) Show your work.

Calculating Change

Applying Percents When you buy a company's stock, you buy a small piece of the company. The amount you have to pay for a stock varies from day to day or even minute to minute. You can make money by selling the stock for more than you paid for it. You can lose money by selling it for less than you paid for it.



Since 1870, when continuous stock trading began at the New York Stock Exchange, the ringing of a bell signals the beginning (9:30 A.M. EST) and ending (4:00 P.M. EST) of trading each day.



24s15.96

Activity

- 1. Copy the table. Add another column for the percent of change in price from Monday to Friday. Indicate whether the percent of change is an increase or a decrease.
- 2. Suppose you and a friend buy stock on Monday and sell it on Friday. You buy 50 shares of Pat's Pastas and 25 shares of Leaping Lizards. Your friend buys 35 shares of Map Makers Plus and 25 shares of Wondermarts. Who made the better investment, you or your friend? Justify your answer.
- 3. Suppose you had \$500 to invest on Monday.
 - a. Which stock(s) would you buy on Monday, and how many shares of the stock(s) wo
 - b. You sell all your shares on Frida
- 4. a. Research Find the price of a stock in the newspaper or online. Find the price at least one day later.
 - b. Writing in Math Would you invest in the company? Explain.

Company	Price per Share on Monday	Price per Share on Friday
All-Star Adventures	\$6.25	\$5.75
Fun Foods, Inc.	\$28.00	\$26.60
Leaping Lizards	\$19.50	\$23,40
← Map Makers Plus	\$13.00	\$14.95
Nature's Nest	\$3.80	\$4.20
Pat's Pastas	\$5.65	\$4.45
Wondermarts	\$34.00	\$32.75
		I GA
ıld you buy? Explain. ay. How much money		7784

POS is the name of the stock.

\$19.14 is the last traded price of the stock.

3s means 300 shares sold.

Stock Trader

A stock trader places orders or buys and sells securities or commodities. A successful trader is very interested in numbers and in investing and understands economic trends.

Stock Certificates

Most securities transactions today are done without stock certificates, although some people like to have the actual certificate in their possession.

or buys ditties, A

Stock Market Floor

Runners at the Chicago Mercantile Exchange wear yellow jackets for visibility. They take orders to buy and sell stocks into and out of the trading areas.





Where You've Been

- In Chapter 2, you learned how to simplify variable expressions and to solve one-step equations and inequalities by adding, subtracting, multiplying, or dividing.
- In Chapter 3, you learned how to solve equations by adding, subtracting, multiplying, or dividing decimals.
- In Chapter 5, you learned how to solve equations by adding, subtracting, or multiplying fractions.



Diagnosing Readiness (For help, go to the lesson in green.)

Instant self-check online and on CD-ROM

Writing Variable Expressions (Lesson 1-1)

Write a variable expression for each situation.

- **1.** three more than p points
- **3.** the number of months in y years
- **5.** twice as many as b baskets
- 2. six fewer than q questions
- **4.** the value in cents of d dimes
- **6.** eight fewer than *n* nickels

Simplifying Expressions (Lesson 2-3)

Simplify each expression.

7.
$$3n + n$$

8.
$$5b + 10 - 8b$$

8.
$$5b + 10 - 8b$$
 9. $12c + 9 + 7c + 4$

10.
$$3x + 2y - 7y - 10x$$
 11. $2(a + 3)$

12.
$$5(m-7)+4m$$

Solving Equations (Lessons 2-5, 2-6)

Solve each equation.

13.
$$a - 3 = 8$$

13.
$$a - 3 = 8$$
 14. $-9 = 12 + x$ **15.** $\frac{m}{7} = -14$ **16.** $-10 = -2b$ **17.** $y \div 2 = 4$ **18.** $6.8 = c - 2.2$ **19.** $\frac{x}{-4} = 8$ **20.** $-40 = 5a$

15.
$$\frac{m}{7} = -14$$

16.
$$-10 = -2l$$

17.
$$v \div 2 = 4$$

18.
$$6.8 = c - 2.2$$

19.
$$\frac{x}{-4} = 8$$

20.
$$-40 = 5a$$

Solving Inequalities (Lessons 2-9, 2-10)

Solve and graph each inequality.

21.
$$c + 6 \ge 7$$

21.
$$c + 6 \ge 7$$
 22. $y - 8 < -6$ **23.** $5b < 20$ **24.** $-3x < 0$

23.
$$5b < 20$$

24.
$$-3x < 0$$

25.
$$12 \le x + 18$$

25.
$$12 \le x + 18$$
 26. $-\frac{x}{3} \ge -5$ **27.** $b - 15 \le 4$ **28.** $\frac{m}{4} \ge 20$

27.
$$b - 15 \le 4$$

28.
$$\frac{m}{4} \ge 20$$